Topology
HW 9
Due Thursday, Nov 18

1. 7.37

2. Show that the one-point compactification of $\mathbb{Z}^+$ is homeomorphic with the subspace $\{0\} \cup \{1/n : n \in \mathbb{Z}^+\}$ in $\mathbb{R}$.

3. Suppose the assumption that X is Hausdorff is dropped in the definition of a one-point compactification. In this case, the definition of the topology for T_Y must be modified. Specifically, the set C must satisfy an additional requirement. Determine what this is and why it is needed in the proof that T_Y is a topology (you do not need to reproduce the entire proof but only comment on where in the proof the additonl condition is needed).

4. Suppose that X is a nonempty Hausdorff space. Also, given $x_0 \in X$ let $Y = X - \{x_0\}$.

a) Is Y Hausdorff?

b) Give an example that shows that X compact does not mean that Y has to be compact. Also, explain why any example must have X with an infinite number of points.

c) Show that if X is compact then Y is locally compact.

5. Prove that the one-point compactification of X is connected (assuming that X is not compact).

6. Prove that locally compact is a topological property.