Solutions to HW7

5.23 Note that given $a \in A$, $\delta_a = \text{dist}(a, B) > 0$ [otherwise, $a \in \bar{B}$]. With this $B(a, \frac{1}{3}\delta_a)$ is open & disjoint from B. Using a similar argument for B let

$$U_A = \bigcup_{a \in A} B(a, \tfrac{1}{3}\delta_a) \quad \& \quad U_B = \bigcup_{b \in B} B(b, \tfrac{1}{3}\delta_b)$$

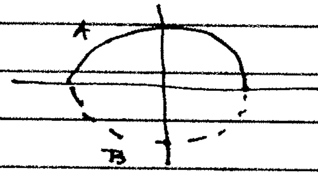
Clearly U_A & U_B are open with $A \subset U_A$ and $B \subset U_B$. It remains to prove that U_A & U_B are disjoint. If not $\exists\, a \in A$ & $b \in B \ni$ $B(a, \frac{1}{3}\delta_a) \cap B(b, \frac{1}{3}\delta_b) \neq \emptyset$. This is not possible given the definitions of δ_a & δ_b ✱

6.1 If not then $\exists$ nonempty & open $U, V \ni X = U \cup V$. Now U open $\Rightarrow U^c$ finite & V infinite with $U^c = V$. This is not possible

6.8 If not then $\exists$ open sets $U, V \ni \bigcup A_j \subset U \cup V$ with $U \cap (\bigcup A_j) \neq \emptyset$ and $V \cap (\bigcup A_j) \neq \emptyset$. Also, $\exists\, j \ni A_j \subset U$ & $A_{j+1} \subset V \Rightarrow A_j \cap A_{j+1} = \emptyset$ ✱

6.18 a) $A: x^2 + y^2 = 1$ & $y \geq 0$
$B: x^2 + y^2 = 1$ & $y \leq 0$
$\Rightarrow$

$A \cap B = \{(-1,0), (1,0)\}$



pf A is connected
$f: [0, \pi] \to A$
$f(\theta) = (\cos\theta, \sin\theta)$
f cont & $[0,\pi]$ connected
$\Rightarrow f[0,\pi] = A$ connected

• Pick any connected A and $B = A \cup \{\text{isolated pt}\}$
$\Rightarrow A \cap B = A$

d) $A = \mathbb{Q} \cap [0,1]$ & $B = \mathbb{I} \cap [0,1] \Rightarrow A \cup B = [0,1]$

6.41 This follows from the th. proved in class that A path connected $\Leftrightarrow \exists\, x_0 \in A \ni x_0$ path connected to every pt. in A (in this case $x_0 = p$ and the paths are straight lines)

6.44 Given $x, y \in A$ then the number of disjoint (except for endpoints) paths from x to y is uncountable. So, only countably many of them can intersect C, and so A is path connected.

6.46 Picking $x_0 \in \bigcap A_\alpha$ and $y, z \in \bigcup A_\alpha \Rightarrow \exists\, \alpha_1, \alpha_2 \ni y \in A_{\alpha_1}$ & $z \in A_{\alpha_2} \Rightarrow$

path from y to z

= path from y to x_0 (in A_{α_1})

+ path from x_0 to z (in A_{α_2})