Solutions to HW5

3.15 Let

$T_{A \times B} \equiv$ top on $A \times B$ as a subspace of $X \times Y$

$T_A \times T_B \equiv$ prod. top. on $A \times B$ (for A, B subspaces)

and let $\mathbb{B}_X, \mathbb{B}_Y$ be basis for X & Y (resp).
In this case we have the following:

1) basis for $X \times Y = \mathbb{B}_X \times \mathbb{B}_Y$ (Th 3.8)

2) basis for $T_{A \times B} = \{ (B_X \times B_Y) \cap (A \times B) : B_X \in \mathbb{B}_X, B_Y \in \mathbb{B}_Y \}$ ← Th 3.5

$= \{ (B_X \cap A) \times (B_Y \cap B) \}$

3) basis for $A = \{ B_X \cap A : B_X \in \mathbb{B}_X \}$ Th 3.5
basis for $B = \{ B_Y \cap B : B_Y \in \mathbb{B}_Y \}$

4) basis for $T_A \times T_B = \{ (B_X \cap A) \times (B_Y \cap B) \}$ Th. 3.8

Given the equality between (2) & (4) the result follows.

3.19 A, B closed $\Rightarrow$ $A \times B$ closed

$$(A \times B)^c = \{ (a,b) : a \notin A \text{ or } b \notin B \}$$

$$= \{ (a,b) : a \notin A \} \cup \{ (a,b) : b \notin B \}$$

$$= \underbrace{(A^c \times Y)}_{\text{open}} \cup \underbrace{\{ X \times B^c \}}_{\text{open}}$$

$(A \times B)^c$ open $\Rightarrow$ $A \times B$ closed

3.20 $\overline{A\times B} = \bar{A} \times \bar{B}$

$(a,b)\in \overline{A\times B} \iff (B_a \times B_b)\cap (A\times B)\neq \emptyset$

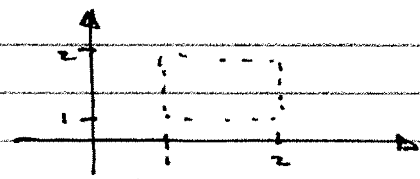
$\forall\, B_a \in \mathcal{B}_A$ & $B_b \in \mathcal{B}_B$ with $a\in B_a$ & $b\in B_b$

$\iff B_a \cap A \neq \emptyset$ and $B_b \cap B \neq \emptyset$

$\iff a\in \bar{A}$ & $b \in \bar{B}$

$\iff (a,b)\in \bar{A}\times \bar{B}$

3.22 a) letting $A = (1,2)\subset \mathbb{R}$ and $B = (1,2)\subset\mathbb{R}$ then $A\times B$ is shown in sketch below



$\partial(A\times B) =$ sides of square

$\partial A \times \partial B = \{0,1\} \times \{0,1\}$

$= \{(0,0),(0,1),(1,0),(1,1)\}$

$=$ vertices of square

b) $\partial(A\times B) = (\partial A\times \bar{B}) \cup (\bar{A}\times\partial B)$

pf: $\overline{A\times B} = \bar{A}\times\bar{B}$

$= (\partial A \cup \mathring{A}) \times (\partial B \cup \mathring{B})$

$= (\partial A\times\partial B)\cup(\partial A\times\mathring{B})\cup(\mathring{A}\times\partial B)\cup\underbrace{(\mathring{A}\times\mathring{B})}_{=(A\times B)^\circ}$

$\Rightarrow$

$\partial(A\times B) = \overline{A\times B} - (A\times B)^\circ$

$= (\partial A\times\partial B)\cup(\partial A\times\mathring{B})\cup(\mathring{A}\times\partial B)$

$= [(\partial A\times\partial B)\cup(\partial A\times\mathring{B})]\cup[(\partial A\times\partial B)\cup(\mathring{A}\times\partial B)]$

$= (\partial A\times\bar{B})\cup(\bar{A}\times\partial B)$

Note the problem stated that $\partial(A\times B)$ be written in terms of $\partial A, \partial B, A,$ & B. The above can be expressed this way by noting that $\bar{B} = B\cup\partial B$ and $\bar{A} = A\cup\partial A$.