Solutions to HW2

1.11 a) No e.g., let $B_1 = (1,3)$ & $B_2 = (2,4)$ $\Rightarrow$ $B_1 \cap B_2 = (2,3)$
and the latter can't contain a subinterval of length 2

b) No e.g., let $B_1 = [1,2]$ & $B_2 = [2,3]$ $\Rightarrow$ $B_1 \cap B_2 = \{2\}$
and the latter can't contain a subinterval of the form $[a,b]$ with $a<b$

c) Yes 1) $x \in \mathbb{R} \Rightarrow x \in [x-1, x+1] \in C_3$
2) $B_1 = [a_1, b_1]$ & $B_2 = [a_2, b_2]$
if $B_1 \cap B_2 \neq \emptyset$ then $B_1 \cap B_2 = [\max\{a_1, a_2\}, \min\{b_1, b_2\}]$
and the latter is the desired B_3

d) Yes 1) $x \in \mathbb{R} \Rightarrow x \in (-x-1, x+1) \in C_4$ (note it's assumed $x \in \mathbb{R}$ replaced with $x \in \mathbb{R}^+$)
2) $B_1 = (-x_1, x_1)$ & $B_2 = (-x_2, x_2)$
$\Rightarrow B_1 \cap B_2 = (-\min\{x_1, x_2\}, \min\{x_1, x_2\})$
and the latter is the desired B_3

e) No e.g., $B_1 = (1,2) \cup \{3\}$ and $B_2 = (2\frac{1}{2}, 3\frac{1}{2}) \cup \{4\frac{1}{2}\}$
$\Rightarrow B_1 \cap B_2 = \{3\}$ and the latter can't contain an open interval (a,b) with $a<b$

1.12 $A = [4,5)$: open, by definition

$B = \{3\}$: not open because can't find interval $[a,b)$, with $a<b$, contained in B

$C = [1,2]$: ~~yes open~~ not open because $[1,2] \cap [2,3) = \{2\}$ and the latter isn't open

$D = (7,8)$: open because $(7,8) = \bigcup_{n=1}^{\infty} [7 + \frac{1}{n}, 8)$

1.15 1) $n \in \mathbb{Z} \Rightarrow n \in A_{n,1}$

2) $A_{a,b} \cap A_{c,d} \neq \emptyset$: $q \in A_{a,b} \cap A_{c,d} \Rightarrow q = a + nb$ & $q = c + md$

$q + \bar{n}bd = a + \bar{n}bd + nb$
$= a + (n + \bar{n}d)b$

$q + \bar{n}bd = c + \bar{n}bd + md$
$= c + (\bar{n}b + m)d$

$\therefore \quad q \in A_{q,b\wedge d} \subset A_{a,b} \cap A_{c,d}$

1.20 Its assumed that $\bigcup_{C_\alpha \in C} C_\alpha = X$ and

$\mathbb{B}_C = \{ B = \cap C_\alpha : \text{finite number of intersections} \}$

checklist:

1) $x \in X \Rightarrow \exists \alpha \ni x \in C_\alpha \in \mathbb{B}_C$ the latter because of the def. of $\mathbb{B}_C$

2) $B_1, B_2 \in \mathbb{B}_C$ & $x \in B_1 \cap B_2$

Now,

$B_1 = \cap C_\alpha$ & $B_2 = \cap \bar{C}_\beta$

$\Rightarrow x \in C_\alpha \;\; \forall \text{ finite } \alpha$ & $x \in \bar{C}_\beta \;\; \forall \text{ finite } \beta$

$\Rightarrow x \in (\cap_\alpha C_\alpha) \cap (\cap_\beta \bar{C}_\beta) \in \mathbb{B}_C$

1.21 given that $\mathbb{B}_C$ contains the individual sets in C it follows that $C \subset T_C$.