Perturbation Methods
MATH-6620
Spring 2012

Instructor: Mark H. Holmes
Email: holmes
Office Hours: MTr 3-4:30
Office: Amos Eaton 322
Phone: 276-6891

Tentative Outline

I. Introduction [1.5 weeks]
- a) asymptotic approximations
- b) algebraic and transcendental equations
- c) regular ODE and PDE expansions

II. Matched Asymptotic Expansions [2 weeks]
- a) boundary layers
- b) interior and corner layers

III. Multiple Scale Methods [2.5 weeks]
- a) two-timing and derivative expansions
- b) applications to nonlinear vibrations and dispersive waves

IV. WKB Method [2.5 weeks]
- a) application to ordinary and partial differential equations
- b) turning points and transition regions
- c) wave propagation
- d) connection with energy methods

V. Homogenization Methods [2 weeks]
- a) periodic and nonperiodic substructures
- b) connection with multiple scales and averaging
- c) derivation of macroscopic descriptions

VI. Bifurcation Problems [3 weeks]
- a) steady states, stability, and Lyapunov-Schmidt procedure
- b) relaxation dynamics
- c) periodic solutions

Required Text: *Introduction to Perturbation Methods (2nd ed)* by M. H. Holmes
Recommended Texts:
1. *Multiple Scale and Singular Perturbation Methods* by Kevorkian and Cole
2. *Advanced Math Methods for Scientists and Engineers* by Bender and Orszag
3. *Perturbation Methods* by Hinch
4. *Perturbations: Theory and Methods* by Murdock
5. *Singular-Perturbation Theory* by Smith

Grading: Homework 100%

Difficulty Level and Pre-requisites
This course requires mathematical maturity and familiarity with the basic solution methods for differential equations (MATH-2400 and 4600). Very little time, if any, will be spent reviewing these background skills and concepts. Also, courses such as MATH-4500 (Methods of PDEs of Math Physics) or MATH-4700 (FOAM) are helpful, but not required.

Course Objectives
We will cover the basic methods involving perturbation approximations. In addition to getting a solid grounding in the fundamentals, it is anticipated that students will gain the following:

- An ability to adapt the methods to a wide variety of problems
- Improved skills at deriving analytical solutions of difficult mathematical problems
- An ability to determine the qualitative behavior for the solutions of nonlinear problems

Activities
On most class days, there will be both lecture and discussions. You will need to read the book to complete your understanding and to prepare for the class discussions.

Academic Integrity
Do not copy or cheat during exams. Before working together on the homework, you must think over the problems on your own. After you have found relevant definitions and theorems and considered several possible approaches to solving a problem, you may work with others. Before you write up your solutions you must separate and rethink and rewrite your assignments alone. You are not allowed to just copy from a shared set of notes. In no case, may you copy from someone else's homework or notes.

All the rules and policies in the Rensselaer handbook should be followed.

Grade Appeals
Due to the nature of proofs, you will need to make sense logically AND advance your argument towards the conclusion to get substantial partial credit. I will grant appeals if I have overlooked something. The appeal must be made within one week of the date the item is returned in class. It is important that you keep all the returned material for the entire semester as they will be your only method for correcting any recording errors that may accidentally occur on my part.

Late Policies
Late homework is usually not accepted without a legitimate excuse. If you have an excuse, you should contact me as soon as possible and I may ask for verification.