Perturbation Methods
HW 5
Due March 26

1. In the study of Raman scattering, one comes across the equation for a forced Morse oscillator with small damping, given as

$$y'' + \varepsilon^2 \alpha y' + (1 - e^{-y})e^{-y} = \varepsilon^3 \cos(1 + \varepsilon^2 \omega)t,$$

where $y(0) = 0$ and $y'(0) = 0$. Also, $\alpha > 0$. Note that this is a slightly modified version of exercise 3.34.

(a) Find a first-term approximation of $y(t)$ that is valid for large t. If you are not able to solve the problem that determines the t_2 dependence, then find the possible steady states (if any) for the amplitude.

(b) Lie and Yuan (1986) used numerical methods to solve this problem. They were interested in how important the value of the damping parameter α is for there to be multiple steady states for the amplitude. They were unable to answer this question because of the excessive computing time it took to solve the problem using the equipment available to them. However, based on their calculations, they hypothesized that multiple steady states for the amplitude are possible even for small values of α. By sketching the graph of A_∞ as a function of ω, for $\alpha > 0$ determine whether or not their hypothesis is correct.

2. The Kirchhoff equation for the transverse vibrations of a weakly nonlinear string is

$$T(t)\partial_x^2 u = \partial_t^2 u, \quad \text{for } 0 < x < 1 \text{ and } 0 < t,$$

where

$$T(t) = 1 + \varepsilon \int_0^1 u_x^2 dx.$$

Also, $u(0, t) = u(1, t) = 0$, $u(x, 0) = \sin(m\pi x)$, and $\partial_t u(x, 0) = 0$. Note that m is a positive integer. Find a first-term approximation of the solution that is valid for large t. Note that this is a modified version of exercise 3.47.

3. 3.54(a)

4. 3.68(b)