Perturbation Methods
HW 3
Due Feb 23

1. Find a composite expansion of the following:

   (a) $\varepsilon y'' + y(y' + 3) = 0$, for $0 < x < 1$, where $y(0) = 1$ and $y(1) = 1$. .

   (b) $\varepsilon y'' - y(y' + 1) = 0$, for $0 < x < 1$, where $y(0) = 3$ and $y(1) = 3$

2. The Reynolds equation from the gas lubrication theory for slider bearings is

$$\varepsilon \frac{d}{dx}(H^3 y y') = \frac{d}{dx}(Hy), \quad \text{for } 0 < x < 1,$$

where $y(0) = y(1) = 1$. Here $H(x)$ is a known, smooth, positive function with $H(0) \neq H(1)$.

   (a) Assuming the boundary layer is at $x = 1$, find a composite expansion of the solution for small ε. Note the boundary layer solution will be defined implicitly but it is still possible to match the expansions.

   (b) (ignore)

3. 2.30(c)

4. Consider the problem of solving

$$\varepsilon y'' = p(y)y' + q(y), \quad \text{for } 0 < x < 1,$$

where $y(0) = y(1) = 1$.

   (a) Where is the layer if $p = e^y$ and $q = 5 + y^2$? You only have to provide a plausible explanation (you do not need to solve anything).

   (b) Where is the layer if $p = -e^y$ and $q = -(1 + y^2)$? You only have to provide a plausible explanation (you do not need to solve anything).