

Algebra

Arithmetic Operations

$$a(b + c) = ab + ac$$

$$\frac{a + c}{b} = \frac{a}{b} + \frac{c}{b}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

Exponents and Radicals

$$x^m x^n = x^{m+n}$$

$$(x^m)^n = x^{mn}$$

$$(xy)^n = x^n y^n$$

$$x^{1/n} = \sqrt[n]{x}$$

$$\sqrt[n]{xy} = \sqrt[n]{x} \sqrt[n]{y}$$

$$\frac{x^m}{x^n} = x^{m-n}$$

$$x^{-n} = \frac{1}{x^n}$$

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

$$x^{m/n} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$$

$$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$$

Factoring Special Polynomials

$$x^2 - y^2 = (x + y)(x - y)$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

Binomial Theorem

$$(x + y)^2 = x^2 + 2xy + y^2 \quad (x - y)^2 = x^2 - 2xy + y^2$$

$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

$$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2}x^{n-2}y^2$$

$$+ \cdots + \binom{n}{k}x^{n-k}y^k + \cdots + nxy^{n-1} + y^n$$

$$\text{where } \binom{n}{k} = \frac{n(n-1) \cdots (n-k+1)}{1 \cdot 2 \cdot 3 \cdots k}$$

Quadratic Formula

$$\text{If } ax^2 + bx + c = 0, \text{ then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Inequalities and Absolute Value

If $a < b$ and $b < c$, then $a < c$.

If $a < b$, then $a + c < b + c$.

If $a < b$ and $c > 0$, then $ca < cb$.

If $a < b$ and $c < 0$, then $ca > cb$.

If $a > 0$, then

$$|x| = a \text{ means } x = a \text{ or } x = -a$$

$$|x| < a \text{ means } -a < x < a$$

$$|x| > a \text{ means } x > a \text{ or } x < -a$$

Geometry

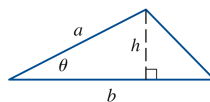
Geometric Formulas

Formulas for area A , circumference C , and volume V :

Triangle

$$A = \frac{1}{2}bh$$

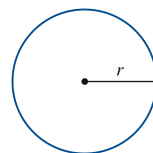
$$= \frac{1}{2}ab \sin \theta$$



Circle

$$A = \pi r^2$$

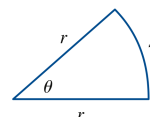
$$C = 2\pi r$$



Sector of Circle

$$A = \frac{1}{2}r^2\theta$$

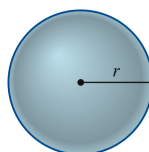
$$s = r\theta \text{ (}\theta \text{ in radians)}$$



Sphere

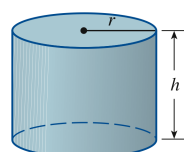
$$V = \frac{4}{3}\pi r^3$$

$$A = 4\pi r^2$$



Cylinder

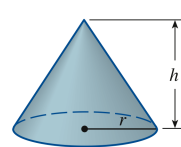
$$V = \pi r^2 h$$



Cone

$$V = \frac{1}{3}\pi r^2 h$$

$$A = \pi r \sqrt{r^2 + h^2}$$



Distance and Midpoint Formulas

Distance between $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{Midpoint of } \overline{P_1P_2}: \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Lines

Slope of line through $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Point-slope equation of line through $P_1(x_1, y_1)$ with slope m :

$$y - y_1 = m(x - x_1)$$

Slope-intercept equation of line with slope m and y-intercept b :

$$y = mx + b$$

Circles

Equation of the circle with center (h, k) and radius r :

$$(x - h)^2 + (y - k)^2 = r^2$$

Trigonometry

Angle Measurement

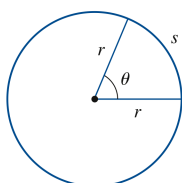
$$\pi \text{ radians} = 180^\circ$$

$$1^\circ = \frac{\pi}{180} \text{ rad}$$

$$1 \text{ rad} = \frac{180^\circ}{\pi}$$

$$s = r\theta$$

(θ in radians)



Right Angle Trigonometry

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

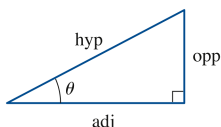
$$\csc \theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\sec \theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\cot \theta = \frac{\text{adj}}{\text{opp}}$$



Trigonometric Functions

$$\sin \theta = \frac{y}{r}$$

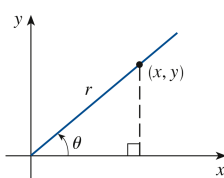
$$\csc \theta = \frac{r}{y}$$

$$\cos \theta = \frac{x}{r}$$

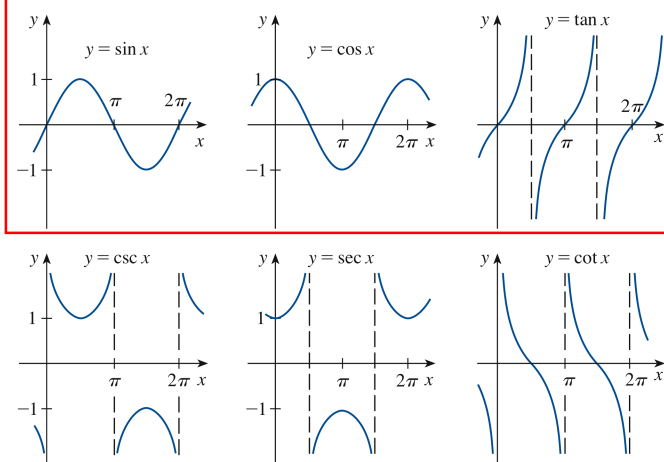
$$\sec \theta = \frac{r}{x}$$

$$\tan \theta = \frac{y}{x}$$

$$\cot \theta = \frac{x}{y}$$



Graphs of Trigonometric Functions



Trigonometric Functions of Important Angles

θ	radians	$\sin \theta$	$\cos \theta$	$\tan \theta$
0°	0	0	1	0
30°	$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
45°	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60°	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
90°	$\pi/2$	1	0	—

Fundamental Identities

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$$

The Law of Sines

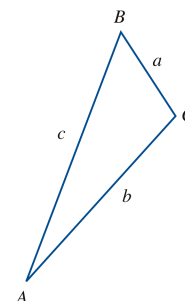
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

The Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$



Addition and Subtraction Formulas

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Double-Angle Formulas

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Half-Angle Formulas

$$\sin^2 x = \frac{1 - \cos 2x}{2} \quad \cos^2 x = \frac{1 + \cos 2x}{2}$$

||| Differentiation Rules

General Formulas

1. $\frac{d}{dx}(c) = 0$
2. $\frac{d}{dx}[cf(x)] = cf'(x)$
3. $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$
4. $\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$
5. $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$ (Product Rule)
6. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$ (Quotient Rule)
7. $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$ (Chain Rule)
8. $\frac{d}{dx}(x^n) = nx^{n-1}$ (Power Rule)

Exponential and Logarithmic Functions

9. $\frac{d}{dx}(e^x) = e^x$
10. $\frac{d}{dx}(a^x) = a^x \ln a$
11. $\frac{d}{dx} \ln |x| = \frac{1}{x}$
12. $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

Trigonometric Functions

13. $\frac{d}{dx}(\sin x) = \cos x$
14. $\frac{d}{dx}(\cos x) = -\sin x$
15. $\frac{d}{dx}(\tan x) = \sec^2 x$
16. $\frac{d}{dx}(\csc x) = -\csc x \cot x$
17. $\frac{d}{dx}(\sec x) = \sec x \tan x$
18. $\frac{d}{dx}(\cot x) = -\csc^2 x$

Inverse Trigonometric Functions

19. $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$
20. $\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$
21. $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$
22. $\frac{d}{dx}(\csc^{-1}x) = -\frac{1}{x\sqrt{x^2-1}}$
23. $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$
24. $\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$

Hyperbolic Functions

25. $\frac{d}{dx}(\sinh x) = \cosh x$
26. $\frac{d}{dx}(\cosh x) = \sinh x$
27. $\frac{d}{dx}(\tanh x) = \text{sech}^2 x$
28. $\frac{d}{dx}(\text{csch } x) = -\text{csch } x \coth x$
29. $\frac{d}{dx}(\text{sech } x) = -\text{sech } x \tanh x$
30. $\frac{d}{dx}(\coth x) = -\text{csch}^2 x$

Inverse Hyperbolic Functions

31. $\frac{d}{dx}(\sinh^{-1}x) = \frac{1}{\sqrt{1+x^2}}$
32. $\frac{d}{dx}(\cosh^{-1}x) = \frac{1}{\sqrt{x^2-1}}$
33. $\frac{d}{dx}(\tanh^{-1}x) = \frac{1}{1-x^2}$
34. $\frac{d}{dx}(\text{csch}^{-1}x) = -\frac{1}{|x|\sqrt{x^2+1}}$
35. $\frac{d}{dx}(\text{sech}^{-1}x) = -\frac{1}{x\sqrt{1-x^2}}$
36. $\frac{d}{dx}(\coth^{-1}x) = \frac{1}{1-x^2}$

Table of Integrals

Basic Forms

1. $\int u \, dv = uv - \int v \, du$
2. $\int u^n \, du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$
3. $\int \frac{du}{u} = \ln |u| + C$
4. $\int e^u \, du = e^u + C$
5. $\int a^u \, du = \frac{a^u}{\ln a} + C$
6. $\int \sin u \, du = -\cos u + C$
7. $\int \cos u \, du = \sin u + C$
8. $\int \sec^2 u \, du = \tan u + C$
9. $\int \csc^2 u \, du = -\cot u + C$
10. $\int \sec u \tan u \, du = \sec u + C$
11. $\int \csc u \cot u \, du = -\csc u + C$
12. $\int \tan u \, du = \ln |\sec u| + C$
13. $\int \cot u \, du = \ln |\sin u| + C$
14. $\int \sec u \, du = \ln |\sec u + \tan u| + C$
15. $\int \csc u \, du = \ln |\csc u - \cot u| + C$
16. $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + C$
17. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$
18. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{u}{a} + C$
19. $\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{u+a}{u-a} \right| + C$
20. $\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right| + C$

Forms Involving $\sqrt{a^2 + u^2}$, $a > 0$

21. $\int \sqrt{a^2 + u^2} \, du = \frac{u}{2} \sqrt{a^2 + u^2} + \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2}) + C$
22. $\int u^2 \sqrt{a^2 + u^2} \, du = \frac{u}{8} (a^2 + 2u^2) \sqrt{a^2 + u^2} - \frac{a^4}{8} \ln(u + \sqrt{a^2 + u^2}) + C$
23. $\int \frac{\sqrt{a^2 + u^2}}{u} \, du = \sqrt{a^2 + u^2} - a \ln \left| \frac{a + \sqrt{a^2 + u^2}}{u} \right| + C$
24. $\int \frac{\sqrt{a^2 + u^2}}{u^2} \, du = -\frac{\sqrt{a^2 + u^2}}{u} + \ln(u + \sqrt{a^2 + u^2}) + C$
25. $\int \frac{du}{\sqrt{a^2 + u^2}} = \ln(u + \sqrt{a^2 + u^2}) + C$
26. $\int \frac{u^2 \, du}{\sqrt{a^2 + u^2}} = \frac{u}{2} \sqrt{a^2 + u^2} - \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2}) + C$
27. $\int \frac{du}{u\sqrt{a^2 + u^2}} = -\frac{1}{a} \ln \left| \frac{\sqrt{a^2 + u^2} + a}{u} \right| + C$
28. $\int \frac{du}{u^2 \sqrt{a^2 + u^2}} = -\frac{\sqrt{a^2 + u^2}}{a^2 u} + C$
29. $\int \frac{du}{(a^2 + u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 + u^2}} + C$