HW 1
Due Jan 28

Instructions: Your assignment must be written up using a word processor (MS Word, LaTeX, etc). The figures should be inserted into the page, with the accompanying text, and the pages numbered.

Rules for Figures: (i) Any graph with more than one curve must contain a legend, and each curve must be easily identifiable using black and white printing. (ii) All axes must be labeled, and the labeling must appear on the printed page at a font size equal to, or slightly larger than, what is used for the text in the write up. (iii) The total width of the figure should be from 70 to 90% of the paragraph width, and be horizontally centered. For example, using 1 in margins then the figure width should be somewhere between 4.55 and 5.85 in. The definition of figure width means the width for the axes and labels, and does not include any margin white space.

1. This problem concerns the Bernoulli equation

$$y' + y^3 = \frac{y}{a+t}\,, \quad \text{for } \ t > 0,$$

where $y(0) = 1$. You are to solve this problem using the backward Euler, trapezoidal, and RK4 methods.
a) Assuming $a = 0.01$, on the same axes plot the three numerical solutions for $0 \le t \le 3$ in the case $M = 80$.
b) Redo (a) for $M = 20$, $M = 40$, and $M = 160$. If one or more of the methods is unstable you can exclude it from the plot (for that value of M) but make sure to state this in your write-up.
c) Compare the four methods based on your results from parts (a) and (b). This includes ease of use, speed of calculation, accuracy of results, and apparent stability.

2. This problem concerns the Oregonator, which consists of the following equations

$$\begin{aligned} \epsilon x' &= \alpha y - xy + x(1-x), \\ \delta y' &= -\alpha y - xy + fz, \\ z' &= x - z, \end{aligned}$$

where $\epsilon = 4 \times 10^{-2}$, $\alpha = 8 \times 10^{-4}$, and $\delta = 4 \times 10^{-4}$. Assume $x(0) = 0.5$, $y(0) = 0$, and $z(0) = 0$.
a) Let $f = 2$. Using ode23s find the solution for $0 \le t \le 3$, and then on the same axes plot x and z as a function of t.
b) Redo (a) using ode45.
c) Any apparent differences between (a) and (b)? Can you explain why one routine takes longer than the other?
d) Redo (a)-(c) for $f = 3$.