particular SPDEs from the point of view of stochastic calculus for Hilbert space–valued stochastic processes.

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Bernt Øksendal
University of Oslo

Composite Asymptotic Expansions. *By Augustin Fruchard and Reinhard Schaefke.* Springer, Heidelberg, 2013. \$49.95. x+161 pp., softcover. Lecture Notes in Mathematics. Vol. 2066. ISBN 978-3-642-34034-5.

Asymptotic solutions to singularly perturbed ordinary differential equations would traditionally be found by matching inner and outer solutions. The result would be a composite asymptotic expansion, i.e., the asymptotic sum of a series in the original variable and another that decays in a stretched variable, both expanded in powers of a root of the given parameter. The second sum provides the nonuniform convergence in a narrow boundary layer, being the difference between the inner expansion and the common part of the inner and outer expansions. Matching can actually be skipped, but do complications arise when turning points occur?

These Alsatian mathematicians provide a convincing case for directly seeking composite asymptotic expansions. One of their main achievements is to realize that one need not have asymptotically exponentially decaying boundary layer corrections—a zero limit with higher order poles should be expected. Like the recent book by Skinner [1], which aimed to justify matching, the authors first convince you that they can succeed in handling difficult linear problems with higher-order turning points. They then consider nonlinear problems in a framework of Gevrey asymptotics. The latter theory builds factorials into the asymptotic estimates, as with Borel transforms (cf. the Hsieh–Sibuya monograph [2] for background material). Applications are made to so-called smooth and nonsmooth canards and to boundary layer resonance.

This important book is carefully written, though not quite an easy read. Learning the details involved will convince you that singular perturbations is actually much more than a casual art form.

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Robert E. O'Malley, Jr.
University of Washington

Introduction to Perturbation Methods. Second Edition. *By Mark H. Holmes.* Springer, New York, 2013. \$79.95. xviii+436 pp., hardcover. ISBN 978-1-4614-5477-9.

This introduction to perturbation methods is a rich, well-written yet subtle textbook. It once again develops the most important techniques needed for singular perturbation problems, including matched asymptotic expansions, multiple scales, and the WKB method for ordinary and for partial differential (and even difference and delay) equations. The outline is much the same as the 1995 first edition, but new material and up-to-date references have been added, including very interesting topics such as weakly coupled linear and nonlinear oscillators, metastability, and the use of Kummer functions to analyze turning point problems. Many wise points are made, many challenging examples and exercises are given, and a wide variety of applications are introduced.

Matching is carried out intuitively and through the use of intermediate variables. Limiting behavior is sought, rather than a few terms of a composite expansion. Using transcendentally small terms to solve the boundary layer resonance problem is one novel and significant approach taken, suggestive of the recent introduction of transseries in purer related mathematics.

Holmes points out how important the elimination of secular terms was to celestial mechanics in the nineteenth century and that this provided the impetus for multiple scale and multiscale methods in twentieth-century analysis and computation. Indeed, he convincingly develops the method of homogenization for important applied problems based on multiple scales, rather than averaging methods. A final chapter on bifurcation and stability adds motivation for further study.

Students and their instructors will benefit greatly from this author's evident broad understanding of applied mathematics and mechanics and his uncommon pedagogical abilities and scholarship. Despite a long list of recent commendable monographs on perturbation methods, Holmes's text will be tough to beat for the ambitious and talented.

ROBERT E. O'MALLEY, JR.
University of Washington

Buoyancy-Driven Flows. *Edited by E. P. Chassignet, C. Cenedese, and J. Verron.* Cambridge University Press, Cambridge, UK, 2012. $120.00. 444 pp., hardcover. ISBN 978-1-107-00887-8.

This new book is composed of 10 chapters originating from the lectures given at the 2010 Alpine Summer School on Buoyancy-driven flows. The lecture slides themselves are available at the conference website: www.to.isac.cnr.it/aosta_old/aosta2010/index.htm, which also provides useful links to the lecturers' backgrounds. A nice introduction precedes the chapters (presumably written by the organizers, although authorship is not acknowledged), which provides an insightful survey of the range of topics covered in the book—although with a strong oceanographic bias—and useful references to the specialized literature.

The book focuses primarily on density-stratified flows that are set into motion by horizontal density variations that are either imposed as initial conditions or created by buoyancy forcing. From the viewpoint of energetics, such horizontal density variations reflect the existence of a finite amount of available potential energy (APE), whose release into kinetic energy (KE) is the primary cause for the fluid flows discussed. Physically, the concept of APE is key to the study of buoyancy-driven motion, because it is the quantity that measures the total amount of work done by buoyancy forces to move the fluid from a notional mechanical equilibrium of minimum potential energy to the actual state considered. Two processes play a central role for understanding the kind of buoyancy-driven motions discussed in this book. The first stems from the fact that as the flow develops, it can become turbulent; this leads to the irreversible dissipation of APE into internal energy because of entrainment/mixing, which can significantly reduce the amount of APE convertible into KE, thus limiting the speed reached by the flow. In numerical models, numerical dissipation arising from discretization errors represents an additional artificial sink of APE that is usually responsible for considerable difficulties in representing some buoyancy-driven flows accurately, as discussed in Chapters 5–7 of this book. The second is due to rotation, which in geophysical flows can considerably slow down the release of APE into KE as the result of the geostrophic adjustment process, which causes the motion to become