Numerical Linear Algebra with Applications S'25
HW 5
Due April 7

1. 4.46 (a), (b), (e) using $\mathbf{A}_4$

2. 4.51

3. (MATLAB) 4.56

4. (MATLAB) In class we approximated a color image by taking the three color planes used to form the image and put them into a columnar matrix in the order $\mathbf{A} = [Red; Green; Blue]$. This exercise explores if the effectiveness of the compression depends on this particular choice. For this problem you need to select a color image. It should be clearly focused, contain a range of colors, and contain at least 10^6 pixels.
a) Apply the approximation procedure used in class to your image. Pick a k so the relative error is in the neighborhood of 0.01. What to hand in for this: a copy of the original image, a copy of image coming from $\mathbf{A}_k$, and the plot of the relative error as a function of k. You should also provide the following information: pixel dimensions of the figure, value of k you picked, and the relative error for your value of k.
b) Apply the approximation procedure to $\mathbf{A} = [Green; Blue; Red]$. What to hand in for this: a copy of the image coming from $\mathbf{A}_k$ (using the same k as in part (a)), the plot of the relative error as a function of k, and the relative error for your value of k. You should comment on what differences or similarities there are between the image and plot with those from part (a).

5. (MATLAB) This exercise illustrates how a SVD can be used to separate, or classify, the voting records for the senators in the 113th Congress. The data table is contained in the file 113.senators.txt. An explanation of the contents of this table, and how to use it with MATLAB, are given in a ReadMe file. You will need to compute the SVD of the voting record data to answer the questions below. Also, this problem is based on a paper by Porter, et al, and a copy can be found on Piazza.
a) Plot $E(k)$ for $1 \leq k \leq 10$. Porter, et al state that "To an excellent approximation, we find that a Representative's voting record can be characterized by just two coordinates." Is this true for your data?
b) The first two columns in $\mathbf{V}$ are $\mathbf{v}_1 = (v_{11}, v_{21}, v_{31}, \cdots)^T$ and $\mathbf{v}_2 = (v_{12}, v_{22}, v_{32}, \cdots)^T$. On the same axes plot the points (v_{i1}, v_{i2}) for the democratic senators (color these points blue), for the independents (color these points green), and for the republican senators (color these red). Note that you should not connect the points with a line (just use markers or symbols). This plot should resemble the plots in Fig. 8 of Porter, et al (but without the names attached to each data point). Also, your axes should be labeled v_1 an v_2.
c) Which variable, v_1 or v_2, can be used to distinguish democrats from republicans? Why? Note that this is the variable Porter, et al call the partisan coordinate, and the other one they call the bipartisan coordinate.
d) Who is the least partisan republican, and who is the least partisan democrat?