Numerical Linear Algebra with Applications S25
HW 4
Due March 24

1. 9.5

2. 9.13(b)

3. 9.21(b)

4. a) The starting point $\mathbf{x}_1$ for the SDM is shown in the figure on next page. Locate on the graph where $\mathbf{x}_2$ and $\mathbf{x}_3$ are located. Also, provide an explanation of how you are able to determine their locations.
b) Locate on the graph where $\mathbf{x}_2$ and $\mathbf{x}_3$ are located if you use the CGM.

5. (MATLAB) Solve $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{A}$ is a matrix obtained as described below. The exact solution should be $\mathbf{x} = (1, 1, \cdots, 1)^T$, and so you should take $\mathbf{b} = \mathbf{Ax}$. You are to solve this using sparse CGM and sparse PCGM. The stopping condition should be: stop if $||\mathbf{x}_k - \mathbf{x}_{k-1}||_\infty < 10^{-8}$ or if $k \geq n$.
a) What is the name of the matrix you picked, the application it comes from (if known), the total number of nonzero entries, and the value of n? Also, submit a plot of the sparsity pattern for your matrix, which you can obtain using the MATLAB command `spy(A)`.
b) Using the Jacobi preconditioner, submit a plot of the error for both methods, and a plot of the iterative error for both methods, as a function of the iteration step.
c) Redo part (b) but use the SSOR preconditioner. In addition to the plots, you must submit a copy of your code.
d) Which method is faster (as measured using computing time)?

Selecting and Importing a Matrix:
1) Visit the SuiteSparse Matrix Collection at https://sparse.tamu.edu. Using the Filters menu, find a real, symmetric, positive definite matrix with $n \geq 20000$. Download the Matrix Market format version. After you untar the file you can open it using a text editor if you are interested in the contents. **Important:** the one you select must be on the "permitted matrices list" on Piazza, and you must tell the instructor of your choice (this is explained in the Piazza post). The reason for this is that everyone is going to be using a different matrix.
2) Download the file *mmread.m* from our Piazza/Resources page. You can then read your matrix into MATLAB using the command `A = mmread('filename.mtx');` Note that $\mathbf{A}$ will be stored as a sparse matrix in MATLAB.

Suggestion: PCGM contains the step $\mathbf{Mz} = \mathbf{r}$, which in MATLAB you could code as $\mathbf{z} = \mathbf{M}\backslash\mathbf{r}$. However, for SSOR it is better to notice that $\mathbf{M} = \overline{\mathbf{L}}\,\overline{\mathbf{U}}$, where $\overline{\mathbf{L}}$ is lower triangular and $\overline{\mathbf{U}}$ is upper triangular. So, it is recommended (but not required) that you use: $\mathbf{y} = \overline{\mathbf{L}}\backslash\mathbf{r}$ and $\mathbf{z} = \overline{\mathbf{U}}\backslash\mathbf{y}$.

Warning: these matrices are typical real-world matrices, and they very likely will misbehave. If you can't make it behave, then you might consider asking to use a different one from the list.

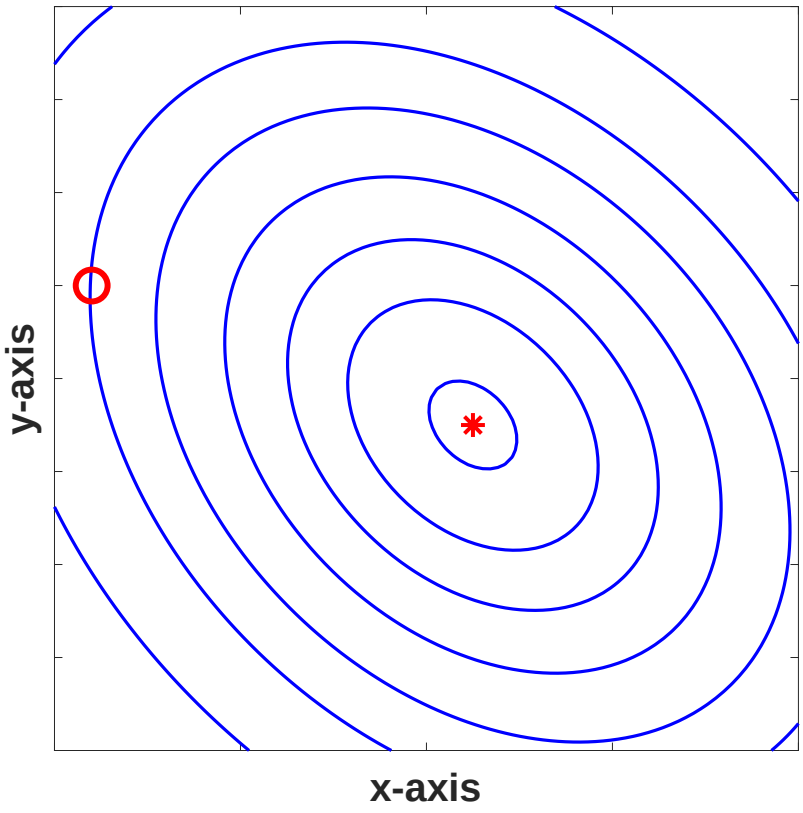

Figure 1: Contour plot for Exercise 4. The $*$ designates the location of the minimum.