

Viscous Flow Simulation of a Two-Dimensional Channel Flow with Complex Geometry Using the Grid-Particle Vortex Method

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Abstract

To avoid a large consumption of computer time instead of the direct vortex methods with Biot-Savart interaction law, the vortex in cell method was used. The local velocity of a particle was obtained by the solution of the Poisson equation for the stream function, its differentiation, and then interpolation of velocity to the vortex particle position from the grid nodes. The Poisson equation for the stream function was solved by fast elliptic solvers. To be able to solve the Poisson equation in a region with a complex geometry, the capacitance matrix technique was used. The viscosity of the fluid was taken in a stochastic manner. A suitable stochastic differential equation was solved by the Huen method. The non-slip condition on the wall was realized by the generation of the vorticity. The program was tested by solving several flows in the channels with a different geometry and at a different Reynolds number. Here we present the testing results concerning the flow in a channel with sudden symmetric expansion.

1 Introduction

The vortex method has been used for the modelling of fluid dynamics phenomena for a long time. Rosenhead (1931) is regarded as the first person to use the point vortex for the numerical study of Kelvin - Helmholtz instability. After him there were many other researchers who used the vortex elements and their ingenuity to study different fluid dynamics phenomena [12,13]. However only Chorin's (1973) work relating to the simulation of viscous flow by the vortex blob method gave great impetus to the investigation and development of vortex methods. The algorithm was designed in such a way that the calculations were carried out in Lagrangian variables and was grid free. A viscous splitting algorithm was used. The solution was obtained in two steps. In the first step, the Euler equation was solved, in the second, the diffusion equation was solved. The local velocity of the blobs was calculated on the basis of the Biot-Savart law. That led to the direct summation of all the velocities induced by the vortex particles present in the flow. Due to the fact that, the number of operations in each time step is proportional to $\sim O(N^2)$, N - number of the particles, computational time is remarkably large. Another approach for velocity calculation of the vortex particles is used in a mixed Eulerian-Lagrangian scheme called the vortex in cell method (VIC). The VIC algorithm was introduced by Christiansen (1973). He demonstrated the usefulness of the method by studying the vortex phenomena of the inviscid flow. Velocities of the vortex particles were obtained through the solution of the Poisson equation for streamfunction by the finite difference method. The velocity of the vortex particles are calculated by the interpolation of the velocities in the grid nodes to the particle position. The VIC method with

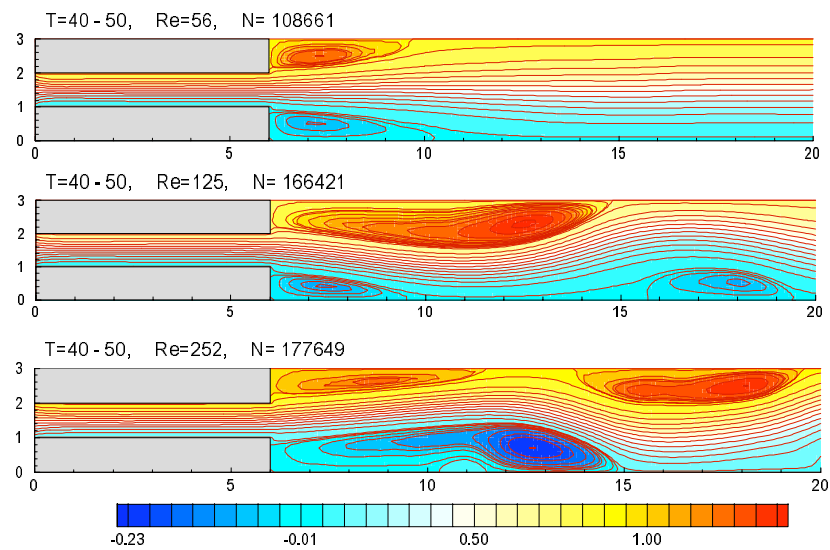


Fig.4 The field of streamfunction values with streamlines at $Re=56, Re=125, R=252, W_0/W=1/3$.

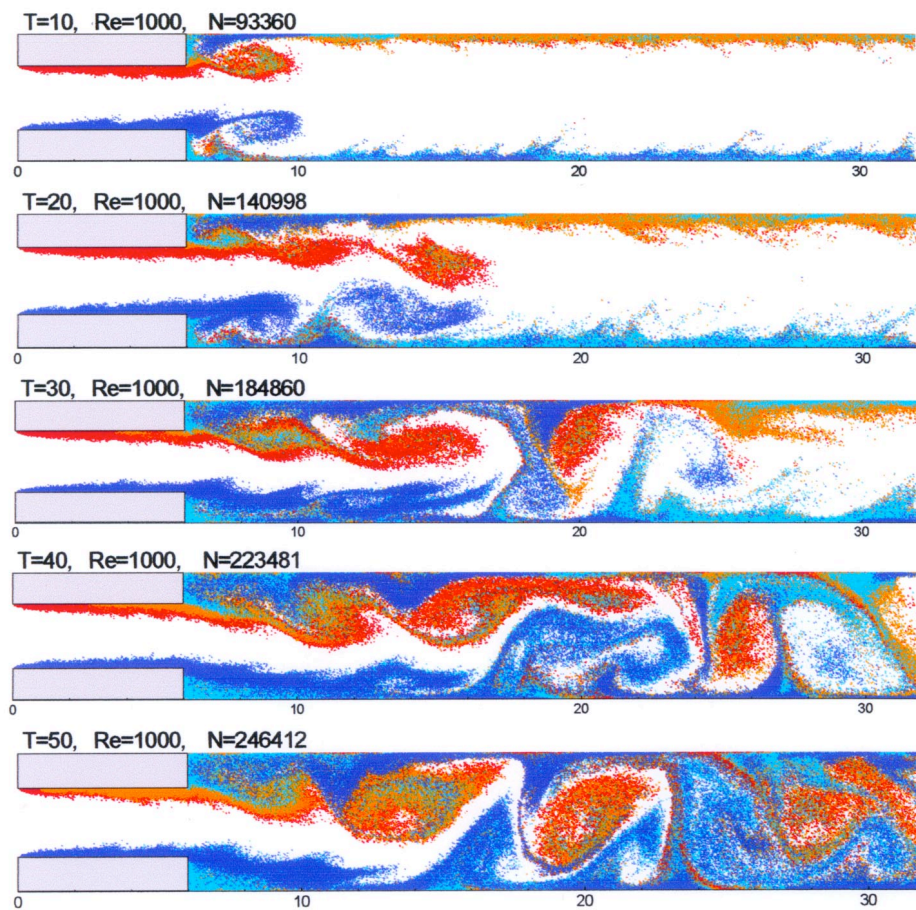


Fig 5. The sequence of vortex particle positions at $Re=1000, W_0/W=1/2$.