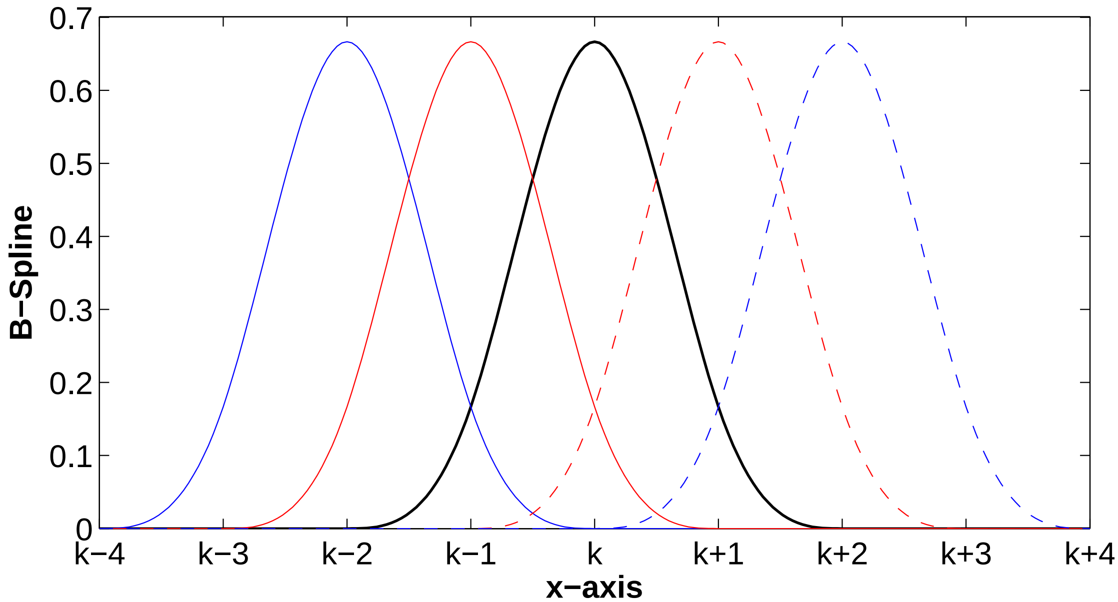
Tridiagonal Matrix Algorithm

$$
\begin{array}{ll}
\text{Set:} & w = a_1,\ y_1 = \dfrac{z_1}{w} \\
\text{For} & i = 2, 3, \cdots, N \\
 & v_i = \dfrac{c_{i-1}}{w} \\
 & w = a_i - b_i v_i \\
 & y_i = \dfrac{z_i - b_i y_{i-1}}{w} \\
\text{End} & \\
\text{For} & j = N-1, N-2, \cdots, 1 \\
 & y_j = y_j - v_{j+1} y_{j+1} \\
\text{End} &
\end{array}
$$

Algorithm for solving $\mathbf{Ay} = \mathbf{z}$ when $\mathbf{A}$ is the tridiagonal matrix

$$
\mathbf{A} = \begin{pmatrix}
a_1 & c_1 & & & & \\
b_2 & a_2 & c_2 & & 0 & \\
 & b_3 & a_3 & c_3 & & \\
 & & \ddots & \ddots & \ddots & \\
 & 0 & & & & c_{N-1} \\
 & & & & b_N & a_N
\end{pmatrix}.
$$

Cubic B-splines



Plot of cubic B-splines $B_{k-2}(x), B_{k-1}(x), B_k(x), B_{k+1}(x), B_{k+2}(x)$. The black curve is $B_k(x)$. On the x-axis, the point x_k is labeled as k (and the same holds for the other x_i's).

$$B_k(x) = \begin{cases} 0 & \text{if} \quad x \le x_{k-2}, \\ \dfrac{1}{6h^3}(x - x_{k-2})^3 & \text{if} \quad x_{k-2} \le x \le x_{k-1}, \\ \dfrac{1}{6} + \dfrac{1}{2h}(x - x_{k-1}) & \\ \quad + \dfrac{1}{2h^2}(x - x_{k-1})^2 - \dfrac{1}{2h^3}(x - x_{k-1})^3 & \text{if} \quad x_{k-1} \le x \le x_k, \\ \dfrac{1}{6} - \dfrac{1}{2h}(x - x_{k+1}) & \\ \quad + \dfrac{1}{2h^2}(x - x_{k+1})^2 + \dfrac{1}{2h^3}(x - x_{k+1})^3 & \text{if} \quad x_k \le x \le x_{k+1}, \\ -\dfrac{1}{6h^3}(x - x_{k+2})^3 & \text{if} \quad x_{k+1} \le x \le x_{k+2}, \\ 0 & \text{if} \quad x_{k+2} \le x. \end{cases}$$

	x_{k-1}	x_k	x_{k+1}	$x_{k\pm m}$ for $m > 1$
B_k	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$	0
B'_k	$\frac{1}{2h}$	0	$-\frac{1}{2h}$	0
B''_k	$\frac{1}{h^2}$	$-\frac{2}{h^2}$	$\frac{1}{h^2}$	0

Table 1: Values of the B-spline $B_k(x)$ at the grid points used in its construction.