Advection Equation Solvers

Methods for solving

$$\frac{\partial u}{\partial t} + a \frac{\partial u}{\partial x} = 0$$

using

$$u_{i,j+1} = Au_{i+1,j} + Bu_{i,j} + Cu_{i-1,j}$$

Method	Coefficients	Truncation Error	CFL	Stability	Mono
Upwind	$A = 0$ $B = 1 - \lambda$ $C = \lambda$	O(h) + O(k)	$\lambda \le 1$	Conditional: $\lambda \leq 1$	Yes
Lax– Friedrichs	$A = \frac{1}{2}(1 - \lambda)$ $B = 0$ $C = \frac{1}{2}(1 + \lambda)$	$O(\frac{h^2}{k}) + O(k) + O(h^2)$	$\lambda \le 1$	Conditional: $\lambda \leq 1$	Yes
Lax– Wendroff	$A = -\frac{\lambda}{2}(1-\lambda)$ $B = 1 - \lambda^{2}$ $C = \frac{\lambda}{2}(1+\lambda)$	$O(h^2) + O(k^2)$	$\lambda \le 1$	Conditional: $\lambda \leq 1$	No
Centered	$A = -\frac{\lambda}{2}$ $B = 1$ $C = \frac{\lambda}{2}$	$O(h^2) + O(k)$	$\lambda \le 1$	Unstable	No

Table 1: Explicit finite difference methods for solving the advection equation, assuming a>0 and $\lambda=ak/h$.