Appendix F
Answers to Selected Exercises

Introduction to the FOAM (2nd Edition)
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Chapter 1

1.3
b) water $v = 0.25$

1.4
b) $p = \alpha\sqrt{r^3/(mG)}$
c) $\alpha = \pi\sqrt{2/\gamma}$

1.6
a) $\omega = (\sigma/(\lambda^3\rho)^{1/2} F(\lambda^2\rho g/\sigma)$

1.7
a) $h_r = RF(E/(R\rho g), h_0/R)$

1.9
a) $v = \mu/(r\rho)F(pr^2\rho/\mu^2, r/\ell)$

1.14
b) $-\frac{1}{2}\eta F' = \frac{d}{d\eta}(F^3F')$

1.17
b) $\alpha = \mu\ell^2/\tau$, $\beta = p\ell^2/(\tau V)$

1.18
b) $\varepsilon = c/\sqrt{km}$

Chapter 2

2.1
e) $\varepsilon - \varepsilon^2 + \frac{5}{6}\varepsilon^3$
g) $\varepsilon^{1/2}(1 + \frac{3}{2}\varepsilon - \frac{9}{8}\varepsilon^2)$
j) $\varepsilon^{-1}(1 + \frac{1}{2}\varepsilon - \frac{1}{12}\varepsilon^2)$

2.3
c) one of the solutions is $x \sim -1 + \frac{1}{2}e^{-1}\varepsilon - \frac{1}{8}\varepsilon^2 e^{-2}$

2.15
$y \sim e^{-x} + \varepsilon(\frac{3}{8}e^{-3x} + Ae^{-x} - (\frac{3}{8} + A)e^{x})$

2.23
$y \sim (2-x)^{-1/3} - e^{-3x/\varepsilon}$

2.24
$y \sim \frac{1}{2}(1 + x^2 + e^{-x/\varepsilon^2})$

2.34
c) $\frac{\rho g \ell^2}{E}(\frac{x}{\ell}(1 - \frac{1}{2}\frac{x}{\ell}) + \frac{\varepsilon}{4}((1 - \frac{x}{\ell})^4 - 1))$

2.37
b) $t_E \sim 2 + \frac{4}{3}\varepsilon$

Chapter 3

3.4
d) $X_2 = H_2 X_2 = 0$. If $H_2(0) \leq X_2(0) + X(0)$ then $H_2 = 0$, $X = X(0) + 2X_2(0) - 2H_2(0)$, and $HX = 2H_2(0) + HX(0) + 2H_2X_2(0)$. If $H_2(0) \geq X_2(0) + X(0)$ then $X = 0$, etc

3.7
e) requirement is $\frac{k_3}{k_1} \leq N$

3.18
f) requirement is $k_4 k_1 N > (k_3 + k_4)k_2$

3.20
e) $b \sim e^{-\lambda\tau} \int_0^\tau (1+z)^{-2} e^{\lambda z} dz$

3.21
e) $a \sim \frac{\lambda(1+\lambda)}{c + 2\lambda\tau} - \frac{1}{1+\lambda} e^{-(1+\lambda)\tau/\varepsilon}$

3.23
a) e: A two term expansion is $x \sim (1 + \sqrt{2})\alpha + \frac{1}{2}(4 + 3\sqrt{2})\alpha^2$
c) A two term expansion is $x \sim 1/2 - \alpha$

Chapter 4

4.18
b) $u = \frac{1}{2}(\pi D t)^{-1/2} \int_{-\infty}^{\infty} f(s) \exp(-(x - s - ct)^2/(4Dt)) ds$

4.19
b) $u = \frac{1}{2c} \int_{x-ct}^{x+ct} g(s) ds$

Chapter 5

5.7 (b) $\rho(x,t) = f(x+6t)e^t$

Chapter 6

6.3 (c) $u = xt/(\alpha + t)$

6.27 (d) $\psi = \alpha\left(-z + 2\sqrt{1-z} + 3/(2-\sqrt{1-z}) + (11/2)\ln\left(-2+\sqrt{1-z}\right)\right)$ where $z = 2\varepsilon_a$.

Chapter 7

7.3 $U = U_0 \sum_{n=0}^{\infty}[H(ct + A - (2n+1)\ell) - H(ct - A - (2n+1)\ell)]$

7.18
(a) $1/(s-\alpha)^2$ for $Re(s) > \alpha$
(b) $\frac{1}{s}\arctan(1/s)$ for $Re(s) > 1$
(c) $\arctan(1/s)$ for $Re(s) > 0$

7.19 check this answer

$$y(t) = \begin{cases} \frac{1}{4}(t-2)\sin(2t) & \text{if } 0 \leq t < \pi, \\ 0 & \text{otherwise.} \end{cases}$$

7.20 $x = e^{3t}\sin(2\sqrt{2}t)$, $y = 2e^{3t}\sin(2\sqrt{2}t)$

7.21

$$u(0,t) = \begin{cases} T\,\mathrm{erfc}(x/(2\sqrt{Dt})) & \text{if} \quad 0 < t < b, \\ T\left(\mathrm{erf}(x/(2\sqrt{D(t-b)})) - \mathrm{erf}(x/(2\sqrt{Dt}))\right) & \text{if} \quad b \leq t, \end{cases}$$

7.22 $u(t) = f(t) + \int_0^t e^{2(t-r)} f(r)dr$

7.23 $u(t) = \frac{\alpha}{\pi\sqrt{t}}$

7.24 $u(t) = \int_0^t f'(t-r)g(r)dr$

Chapter 9

9.1 (b)

$$u_1 = u_0 \frac{\mu_1 y}{\mu_2(h - h_0) + \mu_1 h_0}$$