Appendix B

Answers

Chapter 1

Section 1.2, pg 4

1a) $y = e^{2t} - 1 \Rightarrow y' = 2e^{2t}$ and $2y + 2 = 2e^{2t} - 2 + 2 = 2e^{2t} \Rightarrow y' = 2y + 2$

1b) $y = te^{-t} \Rightarrow y' = e^{-t} - te^{-t} \Rightarrow y' + y = e^{-t} - te^{-t} + te^{-t} = e^{-t}$

1c) $y = \cos(3t) \Rightarrow y'' = -9\cos(3t) \Rightarrow y'' = -9y$

1d) $y = e^{3t} \Rightarrow y' = 3e^{3t}$ and $y'' = 9e^{3t} \Rightarrow y'' + y' - 3y = 9e^{3t} + 3e^{3t} - 12e^{3t} = 0$

1e) $y = e^t + 1 \Rightarrow y' = e^t$ and $y'' = e^t \Rightarrow y'' + 2y' - 3y = e^t + 2e^t - 3(e^t + 1) = -3$

1f) $y = \frac{1}{1+t} \Rightarrow y' = -1/(1+t)^2 \Rightarrow y' + y^2 = -1/(1+t)^2 + 1/(1+t)^2 = 0$

1g) $y = \tan\left(\frac{1}{3}t + 1\right) \Rightarrow y' = \frac{1}{3}\sec^2\left(\frac{1}{3}t + 1\right)$ and $1 + y^2 = 1 + \tan^2\left(\frac{1}{3}t + 1\right) = \sec^2\left(\frac{1}{3}t + 1\right) \Rightarrow 3y' = 1 + y^2$

1h) $y = \ln(1 + t^2) \Rightarrow y' = 2t/(1 + t^2)$ and $2te^{-y} = 2t\exp(-\ln(1 + t^2)) = 2t/(1 + t^2) \Rightarrow y' = 2te^{-y}$

2a) $r = -2$

2b) $r = 1/3$

2c) none

2d) $r = 0, -4$

2e) $r = -3, 1/2$

2f) $r = 2$

2g) none

2h) $r = 0$

2i) none

2j) none

3a) $r = -2$, $c = 1$

3b) $r = -1$, $c = -1$

3c) $r = 1/3$, $c = 3$

3d) $r = 1$, $c = -1$

3e) $r = -2/5$, $c = -7$

3f) $r = -4$, $c = 3$

4a) $y = ce^{2t} \Rightarrow y' = 2ce^{2t} \Rightarrow y' - 2y = 2ce^{2t} - 2ce^{2t} = 0$

4b) $y = ce^{-t} \Rightarrow y' = -ce^{-t} \Rightarrow y' + y = -ce^{-t} + ce^{-t} = 0$

4c) $y = ce^{4t} \Rightarrow y' = 4ce^{4t} \Rightarrow y' - 4y = 4ce^{4t} - 4ce^{4t} = 0$

4d) $y = ce^{t/3} \Rightarrow y' = \frac{1}{3}ce^{t/3} \Rightarrow 3y' - y = ce^{t/3} - ce^{t/3} = 0$

5a) $y = c_1e^{2t} + c_2e^t \Rightarrow y' = 2c_1e^{2t} + c_2e^t$ and $y'' = 4c_1e^{2t} + c_2e^t \Rightarrow y'' - 3y' + 2y = 4c_1e^{2t} + c_2e^t - 3(2c_1e^{2t} + c_2e^t) + 2(c_1e^{2t} + c_2e^t) = c_1(4e^{2t} - 6e^{2t} + 2e^{2t}) + c_2(e^t - 3e^t + 2e^t) = 0$

5b) $y = c_1e^{2t} + c_2e^{-t} \Rightarrow y' = 2c_1e^{2t} - c_2e^{-t}$ and $y'' = 4c_1e^{2t} + c_2e^{-t} \Rightarrow y'' - y' - 2y = 4c_1e^{2t} + c_2e^{-t} - (2c_1e^{2t} - c_2e^{-t}) - 2(c_1e^{2t} + c_2e^{-t}) = c_1(4e^{2t} - 2e^{2t} - 2e^{2t}) + c_2(e^{-t} + e^{-t} - 2e^{-t}) = 0$

5c) $y = c_1e^{-t} + c_2 \Rightarrow y' = -c_1e^{-t}$ and $y'' = c_1e^{-t} \Rightarrow y'' + y' = c_1e^{-t} - c_1e^{-t} = 0$

5d) $y = c_1e^{-t}\cos(2t) + c_2e^{-t}\sin(2t) \Rightarrow$
$y' = (-c_1 + 2c_2)e^{-t}\cos(2t) + (-2c_1 - c_2)e^{-t}\sin(2t)$ and
$y'' = (-3c_1 - 4c_2)e^{-t}\cos(2t) + (4c_1 - 3c_2)e^{-t}\sin(2t) \Rightarrow$
$y'' + 2y' + 5y = [-3c_1 - 4c_2 + 2(-c_1 + 2c_2) + 5c_1]e^{-t}\cos(2t) + [4c_1 - 3c_2 + 2(-2c_1 - c_2) + 5c_2]e^{-t}\sin(2t) = 0$

6a) (i) $y = c_1(-1 + t) \Rightarrow y' = c_1$. So the DE $\Rightarrow c_1 = \frac{t}{1-c_1+c_1t}$. For the RHS to be constant it's required that $c_1 = 1$. (ii) $y = c_1(-1 + t) + c_2(-1 - t)$ $\Rightarrow y' = c_1 - c_2$. So the DE $\Rightarrow c_1 - c_2 = \frac{t}{1-c_1-c_2+(c_1-c_2)t}$. For the RHS to be constant it's required that $c_1 + c_2 = 1$. So the DE $\Rightarrow (c_1 - c_2)^2 = 1 \Rightarrow c_1 - c_2 = \pm 1$. If $c_1 - c_2 = 1$ (and $c_1 + c_2 = 1$), then $c_1 = 1$ and $c_2 = 0$, while if $c_1 - c_2 = -1$, then $c_2 = 1$ and $c_1 = 0$.

6b) (i) $y = c_1(\frac{1}{4}t^2 + t) \Rightarrow y' = c_1(\frac{1}{2}t + 1)$. So the DE $\Rightarrow c_1(\frac{1}{2}t + 1) = \sqrt{1 + c_1(\frac{1}{4}t^2 + t)}$. Taking $t = 0$ it follows that $c_1 = 1$.
(ii) $y = c_1(\frac{1}{4}t^2 + t) + c_2(\frac{1}{4}t^2 + 2t + 3)$. The DE $\Rightarrow c_1(\frac{1}{2}t + 1) + c_2(\frac{1}{2}t + 2) = \sqrt{1 + c_1(\frac{1}{4}t^2 + t) + c_2(\frac{1}{4}t^2 + 2t + 3)}$. Taking $t = 0$ gives $c_1 + 2c_2 = \sqrt{1 + 3c_2}$. Also, differentiating the DE yields $c_1 + c_2 = 1$. So, $1 + c_2 = \sqrt{1 + 3c_2}$. Squaring and solving yields the 2 solutions $c_2 = 0$ (so $c_1 = 1$) and $c_2 = 1$ (so $c_1 = 0$).

7 row 1: y, t, 1, L, ODE, H; row 2: y, t, 2, L, ODE, IH; row 3: θ, t, 2, NL, ODE, IA; row 4: u, t and x, 2, L, PDE, H; row 5: w, x and t, 4, L, PDE, IH; row 6: S and E, t, 1, NL, ODEs, IH

Chapter 2

Section 2.1, pg 12

1a) $y = (9t + c)^{-1/3}$, and $y = 0$
1b) $y = \pm(2e^{-t} + c)^{-1/2}$, and $y = 0$
1c) $y = -1/(\cos t + c)$, and $y = 0$
1d) $y = 3 \pm \sqrt{t^2/2 + c}$
1e) $y = -\ln(\frac{1}{2}t^2 + 2t + c)$
1f) $y = -\frac{1}{3}\ln[3\ln(t + 1) + c]$
1g) $y = -\frac{1}{4}\ln(2e^{2t} + c)$
1h) $y = -\frac{1}{\ln 2}\ln[t\ln(2) + c]$
1i) $y = \frac{1}{3}[-1 \pm (6t + c)^{-1/2}]$, $y = -\frac{1}{3}$
1j) $y = -2 - 1/(t + c)$ and $y = -2$
1k) $y = 3 - 2/(t + c)$ and $y = 3$
1l) $y = \tan(t/3 + c)$
1m) $y = \ln[\tanh(t^2/2 + c)]$, $y = 0$
1n) $y = \ln(ce^t - 1)$
1o) $y = \pm\sqrt{ce^{t^2} - 1}$
2a) $y(t) = 5\,\frac{1}{\sqrt{150\,t+1}}$
2c) $y(t) = 4 + 7\,t$
2d) $y(t) = (1 + \ln(4 + e^t) - \ln(5))^{-1}$
2e) $y(t) = \ln\left(1/2\,\frac{t^2 e + 2}{e}\right)$
2f) $y(t) = -2 + \sqrt{4 + 2\,t}$
2g) $y(t) = 2\arctan(1 + t)$
2h) $y(t) = 5\left(1 + 4\,e^{5\,t}\right)^{-1}$
2i) $y(t) = \frac{1}{2}\ln\left(e^{-2\,t} + e^2 - 1\right) + t$
2j) $y(t) = \ln\left(t/2 + 1/2\sqrt{t^2 + 4}\right)$
2k) $y = \sin(t)$ for $0 \le t \le \pi/2$, and $y = 1$ for $t > \pi/2$
3a) $q(r) = -\frac{1}{\sqrt{14\,r+1}}$
3c) $h(\tau) = -2 + 4\,e^{\tau/3}$
3d) $h(x) = 6\left(2 + e^{3\,x}\right)^{-1}$
3e) $z(r) = 6\,(1 + 6\,\ln((1 + e^r)/2))^{-1}$
3f) $w(\tau) = 1/2\,\ln\left(1/8\,\tau^4 + 1\right)$
3g) $r(\theta) = 2\,(\theta + 1)^2$
3h) $r(\theta) = -1 + \sqrt{2\,\theta^2 + 1}$
4a) $y - \ln(1 + y) = t + 1 - \ln 2$
4b) $15\,t = y^5 + 5\,y + 6$
4c) $y + \ln(1 + y) = t + 5 + \ln(6)$
4d) $p - e^{-p} = r + 2 - e^{-2}$
5a) $aw' = \sqrt{1 + w^2}$, $w(0) = 0$
5b) $w(x) = \sinh(x/a)$
5c) $y = a\cosh(\frac{x}{a}) + h - a\cosh(\frac{L}{a})$

Section 2.2, pg 18

1a) $y(t) = c\,e^{-3\,t}$
1b) $y(t) = -t/2 - 1/4 + e^{2\,t}c$
1c) $y(t) = -2\,t - 14 + e^{t/4}c$
1d) $y(t) = e^t - 1 + e^{-t}c$
1e) $y(t) = \frac{20\,t - \cos(4\,t) + c}{12\,t + 8}$
1f) $y(t) = \frac{t+c}{t+2}$
1g) $y = -\frac{1}{3} + e^{3t}\int_0^t \sqrt{s}e^{-3s}ds + ce^{3t}$
1h) $y = e^{-t/2}\left(\frac{1}{2}\int_0^t \frac{se^{s/2}}{1+s}ds + c\right)$
2a) $y(t) = -4 + 3\,e^t$
2b) $y(t) = 6t - \frac{3}{2} + \frac{3\,e^{-4t}}{2}$
2c) $y(t) = 2\,e^{-t/5}$
2d) $y(t) = -e^{-t} + 4 - 2\,e^{\frac{t}{2}}$
2e) $y(t) = \frac{-t+10}{5+t}$
2f) $y = -(2/3)e^{-t^2/6}\int_0^t e^{s^2/6}ds$
3a) $q(z) = 2 - 3\,e^{-2\,z}$
3b) $p(x) = -2x + \frac{1}{2} - \frac{e^{-4x}}{2}$
3c) $w(\tau) = e^{2\tau} - e^{\tau/2}$
3d) $z(\tau) = -\tau/4 - \frac{5}{16} + \frac{5\,e^{4\,\tau}}{16}$
3e) $h(x) = \frac{-x+14}{x+7}$
3f) $h(z) = \frac{3\,z-1}{5\,z+1}$
4a) $y_p = -3, y_h = c\,e^{2\,t}$
4b) $y_p = 3\,te^{-t}, y_h = c\,e^{-t}$
4c) $y_p = -3 + 1/13\,e^{2\,t}, y_h = c\,e^{t/7}$
4d) $y_p = \int_0^t e^{-t^2-s^2}ds, y_h = c\,e^{-t^2}$
5a) -2
5b) -1
7a) $y' = -2y + 10$, $y(0) = 1$
7b) $w(t) = 1/\sqrt{5 - 4\,e^{-2\,t}}$
7c) $w(t) = 1/\sqrt{5 - 4\,e^{-2\,t}}$

Section 2.3, pg 28

1a) $N = N_0 e^{-kt}$
1b) $k = \ln(4/3)\,\frac{1}{\text{day}}$
1c) $\frac{\ln(2)}{\ln(4/3)}$ days
2a) $N = N_0 e^{-kt}$
2b) $k = \ln(2)/5730\,\frac{1}{\text{yr}}$
2c) $t = \ln(N_0/N)/k$
2d) either 40 or 39 BC

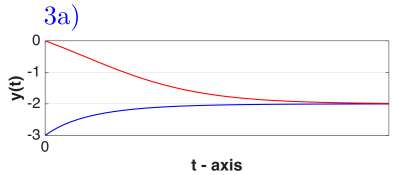
3a) $Q' = -Q/25$, $Q(0) = 200\,\text{g}$
3b) $Q = 200e^{-t/25}$
3c) $50\ln(10)$ min
4a) $Q' = 1/2 - Q/10$, $Q(0) = 0\,\text{g}$
4b) $Q = 5 - 5e^{-t/10}$
4c) $5(1 - e^{-6})\,\text{g}$
5a) $P' = 1000 - P/10$, $P(0) = 0$
5b) $10^4(1 - e^{-1})\,\text{kg}$
6a) $V = 60000 + 10t$
6b) $C' = 50 - 10C/(6000 + t)$, $C(0) = 0$
6c) $C(480) = \frac{324000}{11}\left[1 - \left(\frac{25}{27}\right)^{11}\right]$ lbs
6d) $C' = -C/648$, for $t > 480$, with $C(480)$ from part (b)
7a) $v = -20 + 120e^{-t/2}\,\text{m/s}$
7b) $x = -20t + 240(1 - e^{-t/2})$
7c) $40(5 - \ln 6)\,\text{m}$
8a) $v = -(176/c)(1 - e^{-2ct/11})$
8b) $c = 1\,\text{s}\cdot\text{lbs/ft}$
8c) $v(10) = -176(1 - e^{-20/11})$ fps
8d) $792 + 968e^{-20/11}$ ft
8e) -22 fps
9a) $F_b = Mg$ where $M = \frac{4}{3}\pi a^3\rho$
9b) $A = (M - m)g$
9c) $v = (A/c)(1 - e^{-ct/m})$
9d) $v_T = A/c$; $M < m$
9e) $m/c - cL/A$
10a) $mv' = -mg - cv(1 - \beta v)$, $v(0) = 0$
10b) $v = v_1\frac{1-e^{-rt}}{1-v_1e^{-rt}/v_2}$, $v_1 = (1 - s)/(2\beta)$, $v_2 = (1 + s)/(2\beta)$, $r = (v_2 - v_1)(c\beta/m)$, $s = \sqrt{1 + 4\beta mg/c}$
10c) $(1 - \sqrt{1 + 4\beta mg/c})/(2\beta)$
10d) $v_T = -35.4\,\text{m/s}$ (assuming $g = 9.8$). A fastball is typically in the range of 42 to 47 m/s
11a) $P = \frac{N}{4}\left(4 + z - \sqrt{8z + z^2}\right), z = e^{-rt}$
11b) N
12a) $P = 250\frac{9-e^{-2t}}{3-e^{-2t}}$
12b) 750
13a) $T' = -k(T - 72)$, $T(0) = 200$
13b) $T = 72 + 128e^{-kt}$
13c) $k = \frac{1}{5}\ln(2)\,\frac{1}{\text{min}}$
13d) $5\frac{\ln(64/39)}{\ln 2}$ min
14a) $T' = -k(T - 72)^{5/4}$, $T(0) = 200$
14b) $T = 72 + [4/(kt + 2^{1/4})]^4$
14c) $k = (4^{1/4} - 2^{1/4})/5\,\frac{1}{\text{min}}$
14d) $5\frac{4\cdot 156^{-1/4}-1}{2^{1/4}-1}$ min
15a) $T' = -k(T - 350)$, $T(0) = 70$
15b) $120\frac{\ln(42/37)}{\ln(4/3)}$ min
15c) $120\frac{\ln(1147/693)}{\ln(4/3)}$ min ≈ 3.5 hrs
16a) $-60\ln(1.43)/\ln(0.75)$ min
16b) about 11:45AM
17a) $T' = -[k_0 + k_1(T - T_a)](T - T_a)$
17b) $S(0) = T_0 - T_a$
17d) about 36 sec

Section 2.4, pg 39

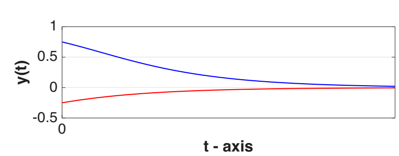
us=unstable; as=asymptotically stable

1a) as
1b) us
1c) as
1d) us
2a) $y = 1$, us; $y = -2$, as
2b) $y = -1$, us; $y = 3$, as
2c) $y = \pm 1$, us; $y = 0$, as
2d) $y = \pm 2$, as; $y = 0$, us
2e) $y = -\ln 2$, as
2f) $y = -2$, as; $y = 2$, us
2g) $y = 0$, as; $y = \ln 3$, us
2j) $y = 0$, as; $y = -2$, as; $y = -1$, us
2i) $y = 0$, us; $y = 1$, as
2j) $y = -2$, us; $y = 3$, as

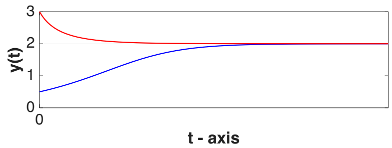
3a)



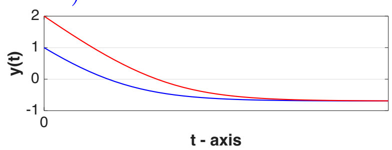
3b)



3c)



3d)



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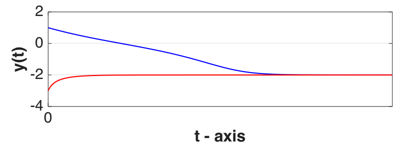

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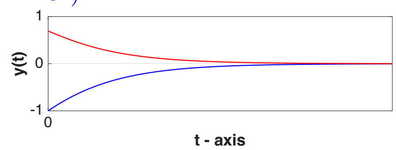
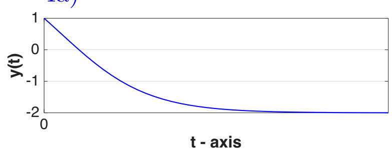
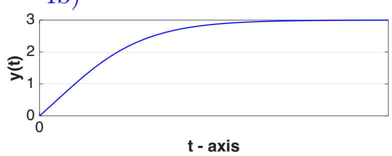


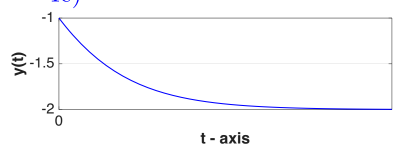
4a)



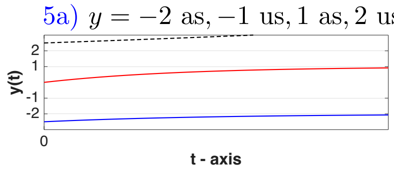
4b)



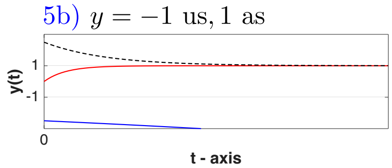
4c)



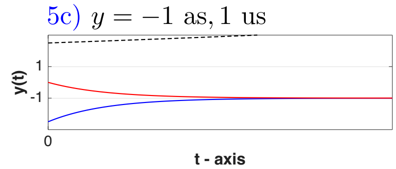
5a) $y = -2$ as, -1 us, 1 as, 2 us



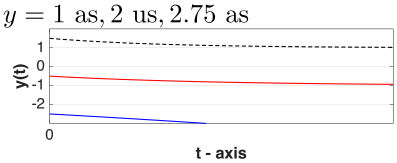
5b) $y = -1$ us, 1 as



5c) $y = -1$ as, 1 us



5d) $y = -2$ us, -1 as, 0 us,
$y = 1$ as, 2 us, 2.75 as



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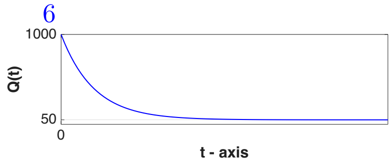
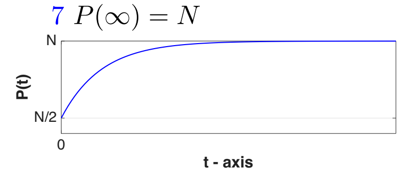


7 $P(\infty) = N$



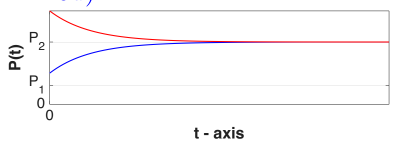
8 $(1 - \sqrt{1 + 4\beta mg/c})/(2\beta)$

10a) $P_1 = \frac{1}{2}N(1-s)$, $P_2 = \frac{1}{2}N(1+s)$, $s = \sqrt{1 - 4h/(rN)}$

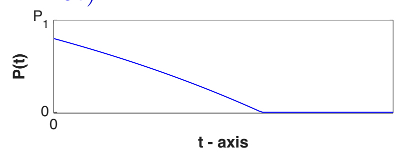
10b) P_1 us, P_2 as

10c) 750

10d)



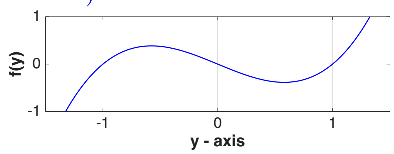
10e)



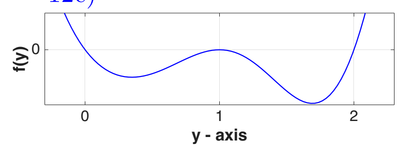
11a) no steady state
11b) $y' < 0$ when $y < 4$
11c) $y' < 0$ when $3 < y < 4$
11d) $y = 2$ is a steady state

12a) not possible; this requires $f(0) = 0$

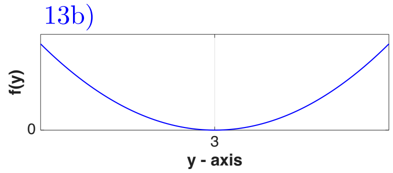
12b)



12c)



13b)



Chapter 3

Section 3.2, pg 46

1 (i) $y_1 = e^{\omega t} \Rightarrow y_1'' = \omega^2 e^{\omega t} = \omega^2 y_1$, and $y_2 = e^{-\omega t} \Rightarrow y_2'' = \omega^2 e^{-\omega t} = \omega^2 y_2$ $\Rightarrow y_1$ and y_2 are solutions. (ii) $W(y_1, y_2) = y_1 y_2' - y_2 y_1' = e^{\omega t}(-\omega e^{-\omega t}) - e^{-\omega t}(\omega e^{\omega t}) = -2\omega \neq 0$. Therefore, y is a general solution.

2 (i) $y_1 = e^{-\alpha t} \Rightarrow y_1'' + 2\alpha y_1' + \alpha^2 y_1 = \alpha^2 e^{-\alpha t} + 2\alpha(-\alpha e^{-\alpha t}) + \alpha^2 e^{-\alpha t} = 0$, and $y_2 = te^{-\alpha t} \Rightarrow y_2'' + 2\alpha y_2' + \alpha^2 y_2 = \alpha^2 te^{-\alpha t} - 2\alpha e^{-\alpha t} + 2\alpha(-\alpha te^{-\alpha t} + e^{-\alpha t}) + \alpha^2 te^{-\alpha t} = 0 \Rightarrow y_1$ and y_2 are solutions. (ii) $W(y_1, y_2) = y_1 y_2' - y_2 y_1' = e^{-\alpha t}(-\alpha te^{-\alpha t} + e^{-\alpha t}) - te^{-\alpha t}(-\alpha e^{-\alpha t}) = e^{-2\alpha t} \neq 0$. Therefore, y is a general solution.

3 (i) $y_1 = 1 \Rightarrow y_1'' + by_1' = 0$, and $y_2 = e^{-bt} \Rightarrow y_2'' + by_2' = b^2 e^{-bt} + b(-be^{-bt}) = 0 \Rightarrow y_1$ and y_2 are solutions. (ii) $W(y_1, y_2) = y_1 y_2' - y_2 y_1' = -be^{-bt} \neq 0$. Therefore, y is a general solution.

4 (i) $y_1 = \cos(\omega t) \Rightarrow y_1'' + \omega^2 y_1 = -\omega^2 \cos(\omega t) + \omega^2 \cos(\omega t) = 0$, and $y_2 = \sin(\omega t) \Rightarrow y_2'' + \omega^2 y_2 = -\omega^2 \sin(\omega t) + \omega^2 \sin(\omega t) = 0 \Rightarrow y_1$ and y_2 are solutions. (ii) $W(y_1, y_2) = y_1 y_2' - y_2 y_1' = \cos(\omega t)(\omega \cos(\omega t)) - \sin(\omega t)(-\omega \sin(\omega t)) = \omega \neq 0$. Therefore, y is a general solution.

5 $W(y_1, y_2) = y_1 y_2' - y_2 y_1' \Rightarrow W' = y_1 y_2'' - y_2 y_1'' = y_1(-py_2' - qy_2) - y_2(-py_1' - qy_1) = p(-y_1 y_2' + y_2 y_1') = -pW$. From (2.23) it follows that $W = W_0 e^{-\int_0^t p(r)dr}$.

Section 3.5, pg 52

1a) $-7, 1$
1b) $-2, 2$
1c) $\frac{1}{2}e^2\sqrt{3}, \frac{1}{2}e^2$
1d) $1 + 2e^2\sqrt{3}, 1 + 2e^2$
1e) $\frac{1}{2}(\sqrt{3} - 1)e^2, \frac{1}{2}(\sqrt{3} + 1)e^2$
1f) $-e^{12}, 0$
3a) $y(t) = c_1 e^{-2t} + c_2 e^t$
3b) $y(t) = c_1 e^{-2t} + c_2 e^{t/2}$
3c) $y(t) = c_1 + c_2 e^{-3t}$
3d) $y(t) = c_1 e^{-t/2} + c_2 e^{t/2}$
3e) $y = c_1 + c_2 t$
3f) $y(t) = c_1 e^{3t} + c_2 e^{3t} t$
3g) $y(t) = c_1 e^{-t/2} + c_2 e^{-t/2} t$
3h) $y = c_1 \sin(t/2) + c_2 \cos(t/2)$
3i) $y(t) = c_1 e^t \sin(t) + c_2 e^t \cos(t)$
3j) $y = e^{-t}(c_1 \sin(2t) + c_2 \cos(2t))$
4a) $y(t) = -1/3\, e^{2t} + 1/3\, e^{-t}$
4b) $y(t) = -8\, e^{\frac{t}{2}} - 2\, e^{-2t}$
4c) $y(t) = -4/3 + 1/3\, e^{-3t}$
4d) $y(t) = 4 - 5\, e^{t/5}$
4e) $y = 3\exp(-(1/3)\sqrt{3}t)$
4f) $y = 5\exp(-(1/2)t)$
4g) $y(t) = -e^{-t} - e^{-t} t$
4h) $y(t) = -1/3 \sin(3t) - \cos(3t)$
4i) $y = -e^{-t} \sin(2t) - e^{-t} \cos(2t)$
4j) $y = 2\, e^{t/2} \cos(t/3)$
5a) $y'' - y = 0$
5b) $y'' - 8y' + 15y = 0$
5c) $y'' - 4y = 0$
5d) $y'' - 2y' = 0$
5e) $y'' - 2y' + y = 0$
5f) $y'' = 0$
5g) $y'' - 4y' + 29y = 0$
5h) $y'' + 4y = 0$
6a) $(1 + t)^3$
6b) $\cos(t^2 + 6t)$
6c) $t + 2$
7a) yes, $y'' + 5y' + 6y = 0$
7b) yes, $y'' + 5y' + 6y = 0$
7c) yes, $y'' + 4y = 0$
8a) -2
8b) $-4, -4, 0$

Section 3.8, pg 61

1a) $y(t) = -e^t$
1b) $y = \frac{-\pi^2 \sin(\pi t) - 3\pi \cos(\pi t) + 2\sin(\pi t)}{\pi^4 + 5\pi^2 + 4}$
1c) $y(t) = -5t^2 - 8t - \frac{42}{5}$

1d) $y(t) = -\frac{3\sin(2t)}{202} - \frac{15\cos(2t)}{101} + 1/6\,e^{-t}$
1e) $y(t) = -4t^3 + 30t^2 - 178t + 535$
1f) $y(t) = \frac{4\cos(2t)}{17} - \frac{33\sin(2t)}{17} - 4$
1g) $y(t) = (5t-2)\,e^t/5$
1h) $y(t) = (3t-1)\cos(3t)/3 - (5t+2)\sin(3t)/5$
1i) $y(t) = t^2 + 4/5\,t + \frac{18}{25}$
1j) $y(t) = 1/10 + 3/13\,e^t$
1k) $y(t) = -t^2 - t^3 - 3/4\,t^4 + 4/3\,t$
1l) $y(t) = -e^t + \frac{3e^{-2t}}{2}$
1m) $y(t) = -\frac{e^{4t}\cos(t)t}{2}$
1n) $y(t) = -\frac{15\cos(t+7)}{37} + \frac{21\sin(t+7)}{37}$
1o) $y(t) = 5 + \frac{\cos(2t)}{2} - \frac{3\sin(2t)}{2}$
1p) $y(t) = -2\sin(2t) - 1/4\cos(2t)$
2a) $y(t) = e^{3t}c_2 + e^{-2t}c_1 - e^t$
2b) $y = c_1e^{-2t} + c_2e^{-t} + \frac{-\pi^2\sin(\pi t) - 3\pi\cos(\pi t) + 2\sin(\pi t)}{\pi^4 + 5\pi^2 + 4}$
2c) $y(t) = e^t c_2 + e^{-5t}c_1 - 5t^2 - 8t - \frac{42}{5}$
2d) $y(t) = \frac{e^{-t}}{6} + 5e^{\frac{t}{5}}c_1 + c_2 - \frac{3\sin(2t)}{202} - \frac{15\cos(2t)}{101}$
2e) $y(t) = e^{-t/3}c_2 + e^{2t}c_1 - 4t^3 + 30t^2 - 178t + 535$
2f) $y(t) = e^{-t/4}c_2 + e^{t/2}c_1 + \frac{4\cos(2t)}{17} - \frac{33\sin(2t)}{17} - 4$
2g) $y(t) = \sin(2t)\,c_2 + \cos(2t)\,c_1 + \frac{(5t-2)e^t}{5}$
2h) $y(t) = c_2e^{-t} + c_1e^{6t} + \frac{(3t-1)\cos(3t)}{3} - \frac{(5t+2)\sin(3t)}{5}$
2i) $y(t) = \sin(2t)\,e^t c_2 + \cos(2t)\,e^t c_1 + t^2 + 4/5\,t + \frac{18}{25}$
2j) $y(t) = e^{-t}\sin(3t)\,c_2 + e^{-t}\cos(3t)\,c_1 + 1/10 + 3/13\,e^t$
2k) $y(t) = 1/3\,e^{3t}c_1 + c_2 - t^2 - t^3 - 3/4\,t^4 + 4/3\,t$
2l) $y(t) = e^{-t}c_2 + e^{2/3\,t}c_1 - e^t + \frac{3e^{-2t}}{2}$
2m) $y(t) = e^{4t}\sin(t)\,c_2 + e^{4t}\cos(t)\,c_1 - \frac{e^{4t}\cos(t)t}{2}$
2n) $y(t) = e^{6t}c_2 + e^{-t}c_1 - \frac{15\cos(t+7)}{37} + \frac{21\sin(t+7)}{37}$
2o) $y(t) = e^{-2t}c_1 + 5 + e^{-t}c_2 + \frac{\cos(2t)}{2} - \frac{3\sin(2t)}{2}$
2p) $y(t) = e^{-t/4}c_1 + c_2 - 2\sin(2t) - 1/4\cos(2t)$
3a) $y(t) = -e^{-2t} + 4e^t - 6t - 3$
3b) $y(t) = -1 + 2\cos(2t) + 2t^2$
3c) $y(t) = -e^t - \sin(t) + \cos(t) + 1$
3d) $y(t) = -\frac{e^{-3t}}{3} + \frac{3t^2}{2} - t + \frac{4}{3}$
3e) $y(t) = -\frac{e^{-2t}}{4} - e^{-2t}t + \frac{e^{2t}}{4}$
3f) $y(t) = e^{\frac{t}{2}} + (1-t)\,e^{-\frac{t}{2}} - 1$
3g) $y(t) = \sin(3t) + \cos(3t) + \cos(3t)\,t$
3h) $y(t) = e^{-t}\sin(2t) + e^{-t}\cos(2t) + e^{-t}$
3i) $y(t) = 2e^{\frac{t}{2}}\sin\left(\frac{t}{2}\right) - 2\sin(t) - \cos(t)$
4a) $y = At^5 + Bt^4 + Ct^3 + Dt^2 + Et + F$
4b) $y = At\cos t + Bt\sin t + C\sin t + D\cos t$
4c) $y = At + B + t(C\cos(2t) + D\sin(2t))$
4d) $y = At + B\sin t + C\cos t$
4e) $y = t(A + Be^{-3t})$
4f) $y = (At^3 + Bt^2 + Ct + D)e^{-2t}$
4g) $y = Ae^{-t}\cos(3t) + Be^{-t}\sin(3t)$
4h) $y = A(t-1)^7 + B(t-1)^6 + C(t-1)^5 + \cdots + G(t-1)^2 + H(t-1) + I$

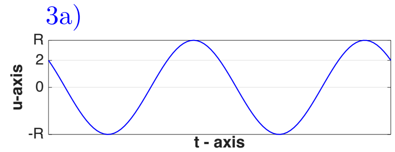
4i) $y = Ate^t \cos t + Bte^t \sin t$
4j) $y = t(A\cos(2t+3) + B\sin(2t+3))$
5a) $y(t) = -2/5\, e^t + e^{6t} c$
5b) $y(t) = e^{-2/3\, t} c + \frac{-3\pi\cos(\pi t) + 2\sin(\pi t)}{9\pi^2 + 4}$
5c) $y(t) = 2/3\, t - 2/9 + e^{-3t} c$
5d) $y(t) = -3t - 15 - 1/6\, e^{-t} + e^{t/5} c$
5e) $y(t) = -\cos(2t)\, t - \frac{2\cos(2t)}{5} - 2\sin(2t)\, t - \frac{3\sin(2t)}{10} + e^{4t} c$
5f) $y(t) = -2t\, e^{-t} - \frac{2\, e^{-t}}{7} - \frac{1}{3} + e^{6t} c$
5g) $y(t) = -1/10\, e^{-t}\cos(t) + 3/10\, \sin(t)\, e^{-t} + e^{-2/3\, t} c$
5h) $y(t) = -1/4\, \cos(2t+5) + 1/4\, \sin(2t+5) + e^{2t} c$

Section 3.9, pg 65

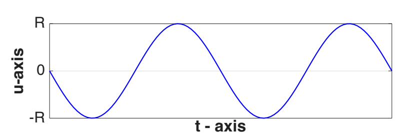
1a) $y_p = -2\, e^{-2t} + 2\, e^{t/2} - 5\, e^{-2t} t$
1b) $y_p = 3 + (-3\cos(t) + 3\sin(t))\, e^t$
1c) $y_p = -e^{-2t}\int_0^t \ln(1+s)\, e^{2s}\, ds + e^t \int_0^t \ln(1+s)\, e^{-s}\, ds$
1d) $y_p = 3t + \frac{2t^{\frac{5}{2}}}{5} - e^{-3t}\int_0^t e^{3s} s^{\frac{3}{2}}\, ds - 1 + e^{-3t}$
1e) $y_p = -2\ln(t+1) + e^{t/5}\int_0^t 2\, \frac{e^{-s/5}}{1+s}\, ds$
1f) $y_p = -e^{-\frac{t}{2}}\int_0^t \sin(s^2+1)\, e^{\frac{s}{2}}\, ds + e^{\frac{t}{2}}\int_0^t \sin(s^2+1)\, e^{-\frac{s}{2}}\, ds$
2a) $y(t) = \frac{4\, e^{\frac{t}{2}}}{5} + \frac{e^{-2t}}{5} + y_p$
2b) $y(t) = -e^t\sin(t) + e^t\cos(t) + y_p$
2c) $y(t) = \frac{e^{-2t}}{3} + \frac{2\, e^t}{3} + y_p$
2d) $y(t) = 1 + y_p$
2e) $y(t) = 1 + y_p$
2f) $y(t) = \frac{e^{\frac{t}{2}}}{2} + \frac{e^{-\frac{t}{2}}}{2} + y_p$
3a) $2t(-t + e^t - 1)$
3b) $1/2\, (t-1)\, e^{2t} + 1/2 + t/2$
3c) $4t^{5/2}$
4b) $\frac{1}{2}\sqrt{t}\sin(t)$

Section 3.10, pg 76

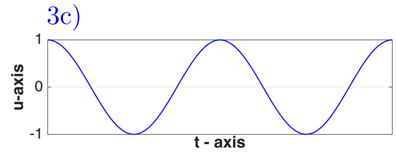
1a) $\omega_0 = 3, R = \sqrt{2}, \varphi = \pi/4$
1b) $\omega_0 = \pi, R = 2, \varphi = -\pi/6$
1c) $\omega_0 = 1, R = 2/\sqrt{3}, \varphi = 2\pi/3$
1d) $\omega_0 = 2, R = 4\sqrt{2}, \varphi = -3\pi/4$
2a) $R = 2, \varphi = \pi/2$
2b) $R = 2, \varphi = -\pi/2$
2c) $R = 2\sqrt{2}, \varphi = \pi/4$
2d) $R = 2\sqrt{2}, \varphi = -3\pi/4$
2e) $R = 2, \varphi = -\pi/3$
2f) $R = 2, \varphi = 2\pi/3$
3a)



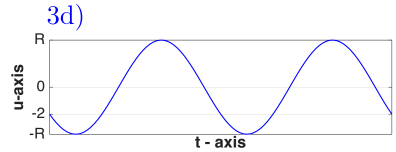
3b)



3c)



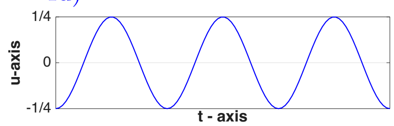
3d)



4a) $u'' + 64u = 0$, $u(0) = -1/4$, $u'(0) = 0$
4b) $u = \frac{1}{4}\cos(8t - \pi)$

4c) $\omega_0 = 8$, $T = \pi/4$, $R = 1/4$

4d)



4e) 1

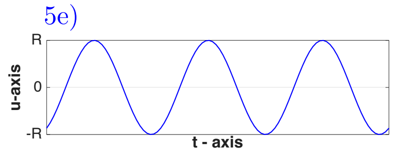
5a) $u'' + 12u = 0$, $u(0) = -1$, $u'(0) = 2$

5b) $u = \frac{2}{\sqrt{3}} \cos\left(2\sqrt{3}t - \frac{5\pi}{6}\right)$

5c) $\omega_0 = 2\sqrt{3}$, $T = \pi/\sqrt{3}$, $R = \frac{2}{\sqrt{3}}$

5d) $t = \pi/(6\sqrt{3})$

5e)



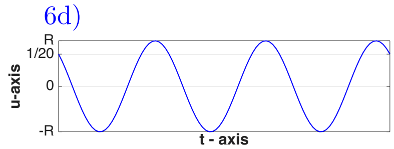
5f) $\sqrt{3}\pi/18$

6a) $u'' + 100u = 0$, $u(0) = 1/20$, $u'(0) = -1/2$

6b) $u = \frac{1}{20}\sqrt{2}\cos\left(10t - \frac{7\pi}{4}\right)$

6c) $\omega_0 = 10$, $T = \frac{\pi}{5}$, $R = \sqrt{2}/20$

6d)



6e) $5(2 + \sqrt{2})$; $3\pi/40$

7 $u'' + \omega_0^2 u = 0$, $\omega_0 = \sqrt{\rho_0 g/(\rho l)}$, $T = 2\pi/\omega_0$

8a) $10\,\mathrm{s}^{-1}$

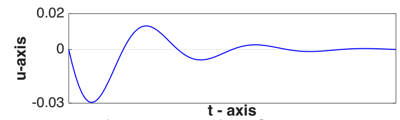
8b) yes, let $u_0' = \sqrt{3}\omega_0 d$

8c) no

9a) $u'' + 4u' + 64u = 0$, $u(0) = 0$, $u'(0) = -1/3$

9b) $u = -\frac{1}{90}\sqrt{15}e^{-2t}\sin(2\sqrt{15}t)$

9c)



9d) $\frac{1}{24}\exp(-\frac{1}{\sqrt{15}}(\frac{3\pi}{2} - \mathrm{Arctan}(\frac{1}{\sqrt{15}})))$

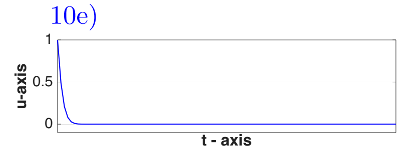
10a) $k = 5$

10b) $c = 3$

10c) $\frac{1}{2}u'' + 3u' + 5u = 0$, $u(0) = 1$, $u'(0) = -2$

10d) $u = \sqrt{2}e^{-3t}\cos(t - \pi/4)$

10e)

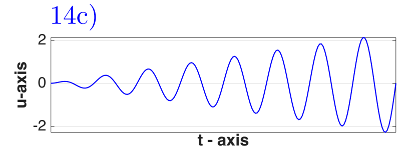


11c) $R = |u_0|\sqrt{1 + (\lambda/\mu)^2}$, for $u_0 \neq 0$, $\varphi = \arctan\left(-\frac{\lambda}{\mu}\right)$, where $0 < \varphi < \pi/2$ if $u_0 > 0$, and $-\pi < \varphi < -\pi/2$ if $u_0 < 0$

14a) $\frac{1}{8}u'' + 32u = 3\cos(16t)$, $u(0) = 0$, $u'(0) = 0$

14b) $u(t) = 3/4 \sin(16\,t)\,t$

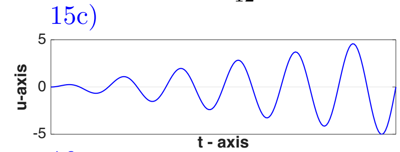
14c)



15a) $2u'' + 18u = 5\cos(3t)$, $u(0) = 0$, $u'(0) = 0$

15b) $u(t) = \frac{5\sin(3\,t)t}{12}$

15c)



16 no

17a) Yes, and the reason is superposition

17b) assuming ω_1 is positive, resonance only if $\omega_1 = 2\omega_0$

Section 3.11, pg 81

1a) $y(x) = c_1\,x^2 + c_2\,x^2\ln(x)$

1b) $y(x) = c_1\,x^3\sin(\ln(x)) + c_2\,x^3\cos(\ln(x))$

1c) $y(x) = \frac{c_1}{\sqrt{x}} + c_2\,\sqrt[3]{x}$

1d) $y(x) = c_1\,\sqrt{x}\sin\left(1/2\,\sqrt{3}\ln(x)\right) + c_2\,\sqrt{x}\cos\left(1/2\,\sqrt{3}\ln(x)\right)$

1e) $y(x) = c_1\,x^2\sin(3\,\ln(x)) + c_2\,x^2\cos(3\,\ln(x))$

1f) $y(x) = \frac{c_1}{x} + \frac{c_2}{x^{2/5}}$

1g) $y(x) = c_2\,\ln(x) + c_1$

1h) $y(x) = c_2\,x^3 + c_1$

1i) $y(x) = \frac{c_1}{x^n} + c_2\,x^{n+2}$

2a) $y(x) = -x^2\mathrm{e} + x\mathrm{e}^x$

2b) $y(x) = -1/4\,x^4 - x + 1/4 + x^2$

2c) $y(x) = 2 - 2\,x + (x + 1)\ln(x)$

2d) $y(x) = 1/4\,x^2 - 3/2\,\ln(x) + 3/4$

2e) $y(x) = \frac{1}{2}\left((x - 1)^{-1} + x - 1\right)$

Chapter 4

Section 4.1, pg 87

1a) $\mathbf{A} = \begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix}$

1b) $\mathbf{A} = \begin{bmatrix} -1/2 & 0 \\ 1/3 & 1/3 \end{bmatrix}$

1c) $\mathbf{A} = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 0 \\ -1 & 5 & 0 \end{bmatrix}$

1d) $\mathbf{A} = \begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix}$, $\mathbf{x}_0 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

1e) $\mathbf{A} = \begin{bmatrix} 2 & 0 & -1 \\ 1 & 1 & 1 \\ 0 & 2/3 & 2 \end{bmatrix}$, $\mathbf{x}_0 = \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix}$

2 i) $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -c/a & -b/a \end{bmatrix}$; *ii*) $\mathbf{a}_1 = \begin{bmatrix} 1 \\ r_1 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 1 \\ r_2 \end{bmatrix}$

a) $a = 1$, $b = 2$, $c = -3$, $r_1 = 1$, $r_2 = -3$; b) $a = 4$, $b = 0$, $c = 1$, $r_1 = \frac{1}{2}i$, $r_2 = -\frac{1}{2}i$; c) $a = 4$, $b = 3$, $c = -1$, $r_1 = 1/4$, $r_2 = -1$ d) $a = 1$, $b = 4$, $c = 0$, $r_1 = 0$, $r_2 = -4$

3a) $\mathbf{x}(0) = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$

3b) $\mathbf{x}(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

3c) $\mathbf{x}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

5c) $\mathbf{x} = c_1 \begin{pmatrix} -1 \\ 2 \end{pmatrix} e^{-3t} + c_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{2t}$

Section 4.3, pg 94

1 a) indep, b) dep, c) dep, d) indep

2a) $r_1 = 3$ with $\mathbf{x}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $r_2 = -2$ with $\mathbf{x}_2 = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$

2a) $r_1 = -1$ with $\mathbf{x}_1 = \begin{pmatrix} -7 \\ 1 \end{pmatrix}$, $r_2 = 5$ with $\mathbf{x}_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

3a) $r_1 = 2 + 2i$ with $\mathbf{x}_1 = \begin{pmatrix} 2i \\ 1 \end{pmatrix}$, $r_2 = 2 - 2i$ with $\mathbf{x}_2 = \begin{pmatrix} -2i \\ 1 \end{pmatrix}$

3a) $r_1 = -1 + 2i$ with $\mathbf{x}_1 = \begin{pmatrix} -3 - 2i \\ 1 \end{pmatrix}$, $r_2 = -1 - 2i$ with $\mathbf{x}_2 = \begin{pmatrix} -3 + 2i \\ 1 \end{pmatrix}$

Section 4.5, pg 101

1a) $\begin{bmatrix} c\_1 \, e^{-3\,t} + c\_2 \, e^{2\,t} \\ -1/3 \, c\_1 \, e^{-3\,t} + 1/2 \, c\_2 \, e^{2\,t} \end{bmatrix}$

1b) $\begin{bmatrix} c\_1 \, e^{-t/2} + c\_2 \, e^{t/2} \\ -2 \, c\_1 \, e^{-t/2} + 2 \, c\_2 \, e^{t/2} \end{bmatrix}$

1c) $\begin{bmatrix} c\_1 + c\_2 \, e^{5\,t} \\ -2 \, c\_1 + 3 \, c\_2 \, e^{5\,t} \end{bmatrix}$

1d) $\left[\begin{array}{c} -c\_2\,e^{2\,t} \\ c\_1\,e^{2\,t} + c\_2\,e^{2\,t}t \end{array}\right]$

1e) $\left[\begin{array}{c} c\_1\,e^{-2\,t} \\ c\_2\,e^{-2\,t} \end{array}\right]$

1f) $\left[\begin{array}{c} 5c\_1\sin(3t) + 5c\_2\cos(3t) \\ c\_1\left(-\sin(3t) + 3\cos(3t)\right) + c\_2\left(-\cos(3t) - 3\sin(3t)\right) \end{array}\right]$

1g) $\left[\begin{array}{c} 2c\_1\sin(4t) + 10c\_2\cos(4t) \\ c\_1\left(\sin(4t) - \cos(4t)\right) + 5c\_2\left(\cos(4t) + \sin(4t)\right) \end{array}\right]$

1h) $\left[\begin{array}{c} c\_1\,e^{t/2}\sin(t) + c\_2\,e^{t/2}\cos(t) \\ c\_1\,\left(-2\,e^{t/2}\sin(t) + 4\,e^{t/2}\cos(t)\right) \\ +c\_2\,\left(-2\,e^{t/2}\cos(t) - 4\,e^{t/2}\sin(t)\right) \end{array}\right]$

1i) $\left[\begin{array}{c} 2c\_1\,e^{-t}\sin(3t) + 2c\_2\,e^{-t}\cos(3t) \\ c\_1\left(e^{-t}\sin(3t) - e^{-t}\cos(3t)\right) + \\ c\_2\left(e^{-t}\cos(3t) + e^{-t}\sin(3t)\right) \end{array}\right]$

1j) $\left[\begin{array}{c} c\_1 \\ c\_2 \end{array}\right]$

2 *a)* $\left\{c\_1 = \frac{18}{5}, c\_2 = \frac{2}{5}\right\}$, *b)* $\{c\_1 = 9/4, c\_2 = 7/4\}$,
c) $\left\{c\_1 = \frac{13}{5}, c\_2 = 7/5\right\}$, *d)* $\{c\_1 = -1, c\_2 = -4\}$, *e)* $\{c\_1 = 4, c\_2 = -1\}$,
f) $\left\{c\_1 = -\frac{1}{15}, c\_2 = \frac{4}{5}\right\}$, *g)* $\left\{c\_1 = 3, c\_2 = \frac{2}{5}\right\}$, *h)* $\{c\_1 = 7/4, c\_2 = 4\}$,
i) $\{c\_1 = 3, c\_2 = 2\}$,
j) $\{c\_1 = 4, c\_2 = 1\}$

3a) $r_1 = 3$ with $\mathbf{x}_1 = \alpha\begin{pmatrix}1\\1\end{pmatrix}$, $r_2 = 1$ with $\mathbf{x}_2 = \alpha\begin{pmatrix}1\\-1\end{pmatrix}$

3b) $r_1 = -5$ with $\mathbf{x}_1 = \alpha\begin{pmatrix}1\\0\end{pmatrix}$, $r_2 = 0$ with $\mathbf{x}_2 = \alpha\begin{pmatrix}1\\3\end{pmatrix}$

3c) $r_1 = -2$ with $\mathbf{x}_1 = \alpha\begin{pmatrix}1\\0\end{pmatrix}$, $r_2 = 4$ with $\mathbf{x}_2 = \alpha\begin{pmatrix}0\\3\end{pmatrix}$

3d) $r_1 = -8$ with $\mathbf{x}_1 = \alpha\begin{pmatrix}1\\0\end{pmatrix}$, $r_2 = -1$ with $\mathbf{x}_2 = \alpha\begin{pmatrix}-1\\3\end{pmatrix}$

4a) $\left[\begin{array}{c} c\_1\,e^{-t} + c\_2\,e^{2\,t} \\ -2\,c\_1\,e^{-t} + c\_2\,e^{2\,t} - c\_3\,e^{-t} \\ c\_1\,e^{-t} + c\_2\,e^{2\,t} + c\_3\,e^{-t} \end{array}\right]$

4b) $\left[\begin{array}{c} c\_1\,e^{t} + 3\,c\_2\,e^{2\,t} + 2\,c\_3\,e^{t}t \\ c\_2\,e^{2\,t} \\ c\_2\,e^{2\,t} + c\_3\,e^{t} \end{array}\right]$

4c) $\left[\begin{array}{c} 2\,c\_1\,e^{t} - c\_2\,e^{-2\,t} \\ 3\,c\_1\,e^{t} \\ 7\,c\_1\,e^{t} + 4\,c\_2\,e^{-2\,t} + c\_3\,e^{-t} \end{array}\right]$

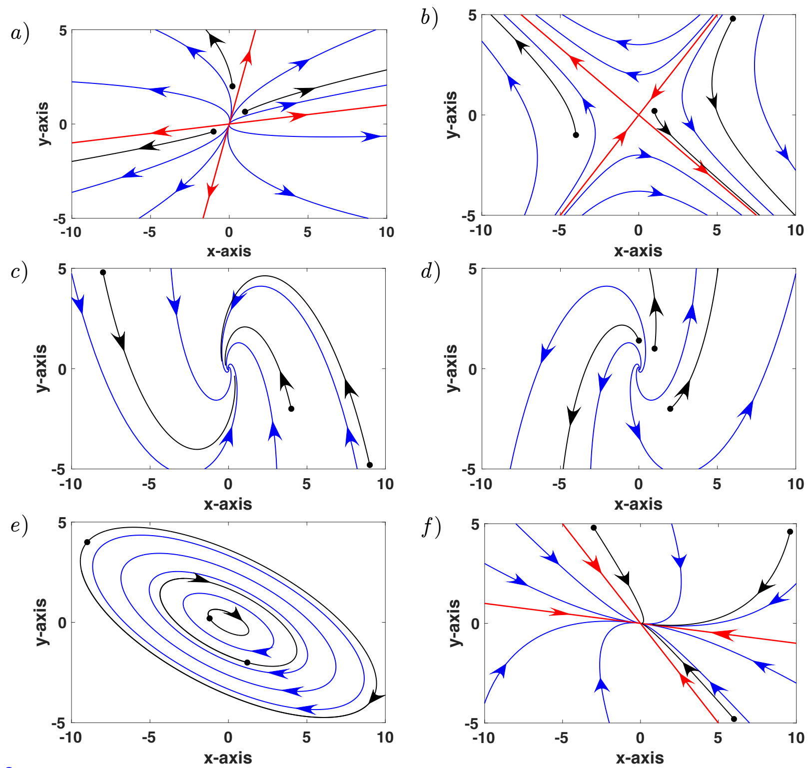
4d) $\left[\begin{array}{c} c\_1\,e^{-t} \\ 2\,c\_2\,e^{\sqrt{5}t} + 2\,c\_3\,e^{-\sqrt{5}t} \\ -c\_3\,\left(\sqrt{5}+1\right)e^{-\sqrt{5}t} + c\_2\,e^{\sqrt{5}t}\left(\sqrt{5}-1\right) \end{array}\right]$

5b) $y_1 = 5 + 5\,e^{-\frac{t}{25}}, y_2 = -5\,e^{-\frac{t}{25}} + 5$

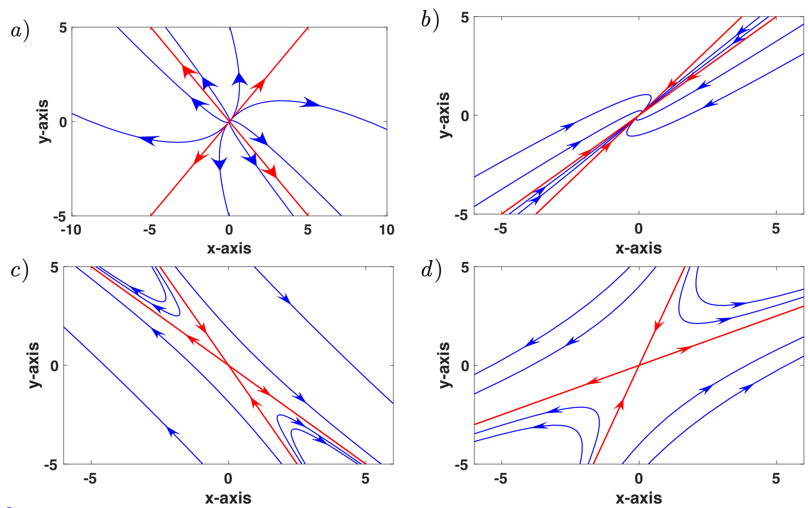
5c) $y_1 = y_2 = 5$

Section 4.6, pg 109

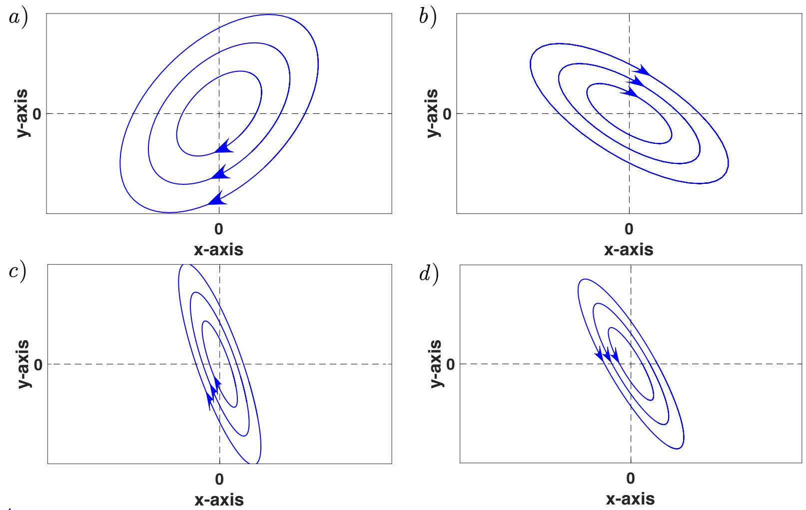
1



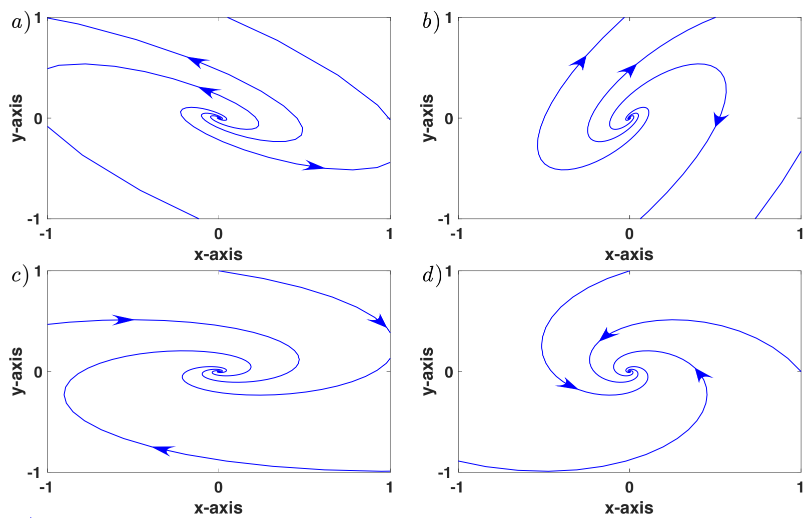
2



3



4



5b) directions reverse

5d) $a > 0$, $c < 0$

6b) $c > 0$ and $c > 0$

Section 4.7, pg 115

us=unstable; as=asymptotically stable; ns=neutrally stable

si=sink; so=source; ssi=spiral sink; sso=spiral source; sa=saddle; c=center

1a) us, sa

1b) as, si

1c) us, so

1d) us, so

1e) as, si

1f) us, sso

1g) as, ssi

1h) ns, c

1i) ns, c

2a) $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -k/m & 0 \end{bmatrix}$, ns

2b) $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -k/m & -c/m \end{bmatrix}$, as

2c) yes

3a) $\mathbf{u}_s = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$, us, sa

3b) $\mathbf{u}_s = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$, as, si

3c) $\mathbf{u}_s = \begin{pmatrix} 1/5 \\ -8/5 \end{pmatrix}$, as, ssi

3d) $\mathbf{u}_s = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, us, sso

5a) us
5b) as
5c) us
5d) us
5e) ns
5f) as

Section 4.8, pg 117

1a) $\mathbf{A} = \begin{pmatrix} 0 & -1/m \\ k & -k/c \end{pmatrix}$, $v(0) = 0,\ f(0) = ku_0$

1b) $v = -\frac{2}{\sqrt{3}} \mathrm{e}^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}\,t}{2}\right)$, $f = \mathrm{e}^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}\,t}{2}\right) - \frac{\mathrm{e}^{-\frac{t}{2}} \sqrt{3}\, \sin\left(\frac{\sqrt{3}\,t}{2}\right)}{3}$

1c) $u = \mathrm{e}^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}\,t}{2}\right) + \frac{\mathrm{e}^{-\frac{t}{2}} \sqrt{3}\, \sin\left(\frac{\sqrt{3}\,t}{2}\right)}{3}$

2a) $\mathbf{K} = \begin{bmatrix} \frac{k1+k2}{m1} & -\frac{k2}{m1} \\ -\frac{k2}{m2} & \frac{k2}{m2} \end{bmatrix}$

2c) $\lambda_1 = 4$ with $\mathbf{a}_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$, $\lambda_2 = 1$ with $\mathbf{a}_2 = \begin{pmatrix} 1/2 \\ 1 \end{pmatrix}$

2d) $\mathbf{u} = \mathbf{a}_1[d_1 \cos(2t) + d_2 \sin(2t)] + \mathbf{a}_2[d_3 \cos(t) + d_4 \sin(t)]$

2e) $u_1 = \frac{\sin(t)}{3} + \frac{\sin(2t)}{3}$, $u_2 = \frac{2\sin(t)}{3} - \frac{\sin(2t)}{3}$

Chapter 5

Section 5.1, pg 124

1a) $\mathbf{y} = \begin{pmatrix} u \\ v \end{pmatrix}$, $\mathbf{f} = \begin{pmatrix} u^2 - v \\ 2u - 3v \end{pmatrix}$

1b) $\mathbf{y} = \begin{pmatrix} u \\ v \end{pmatrix}$, $\mathbf{f} = \begin{pmatrix} u^2 + v^2 \\ \sin(u)/2 \end{pmatrix}$

1c) $\mathbf{y} = \begin{pmatrix} u \\ v \end{pmatrix}$, $\mathbf{f} = \begin{pmatrix} e^u - v \\ uv \end{pmatrix}$, $\mathbf{y}_0 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

1d) $\mathbf{y} = \begin{pmatrix} u \\ v \end{pmatrix}$, $\mathbf{f} = \begin{pmatrix} v \\ -(1-u^2)v - u \end{pmatrix}$

1e) $\mathbf{y} = \begin{pmatrix} u \\ v \end{pmatrix}$, $\mathbf{f} = \begin{pmatrix} v \\ -e^u + 1 \end{pmatrix}$

1f) $\mathbf{y} = \begin{pmatrix} u \\ v \end{pmatrix}$, $\mathbf{f} = \begin{pmatrix} v \\ -u - u^3 \end{pmatrix}$, $\mathbf{y}_0 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

1g) $\mathbf{y} = \begin{pmatrix} S \\ E \end{pmatrix}$, $\mathbf{f} = \begin{pmatrix} -k_1 ES + k_{-1}(E_0 - E) \\ -k_1 ES + (k_2 + k_{-1})(E_0 - E) \end{pmatrix}$, $\mathbf{y}_0 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

1h) $\mathbf{y} = \begin{pmatrix} x \\ y \end{pmatrix}$, $\mathbf{f} = \begin{pmatrix} ax - bxy \\ -cy + dxy \end{pmatrix}$

1i) $\mathbf{y} = \begin{pmatrix} y \\ v \end{pmatrix}$, $\mathbf{f} = \begin{pmatrix} v \\ -\frac{gR^2}{(R+y)^2} \end{pmatrix}$, $\mathbf{y}_0 = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$

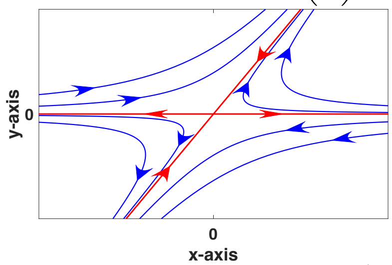
1j) $\mathbf{y} = \begin{pmatrix} r \\ v \end{pmatrix}$, $\mathbf{f} = \begin{pmatrix} v \\ \frac{\alpha^2}{r^3} - \frac{\mu}{r^2} \end{pmatrix}$, $\mathbf{y}_0 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

2a) $(u, v) = (1/2, 0), (1/3, 1/4)$
2b) $(u, v) = (0, 0), (-1, 1)$
2c) $(u, v) = (1/4, 4)$
2d) $(S, P) = (1, 1), (0, 0), (2, 0)$
2e) $(S, I) = (5, 0), (1, 2)$
2f) $(s, c) = (-1, 1)$
2g) $(x, y) = (0, 0)$
2h) $(x, y) = (0, 2), (0, -1), (2, 0)$
2i) $(x, y) = (0, 0), (0, 6), (1, 3), (4, 0)$
2j) $(u, v) = (0, 0)$

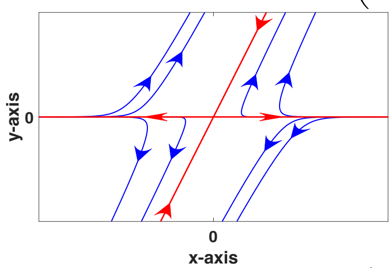
Section 5.2, pg 135

1a) $(u, v) = (1, -1)$, us, sa
1b) $(u, v) = (0, 0)$, us, sa
1c) $(x, y) = (0, 0)$, as, si; $(x, y) = (2/3, 4/9)$, us, sa
1d) $(S, E) = (0, E_0)$, us, sa
1e) $(u, v) = (1/2, 0)$, us, sa; $(1/3, 1/4)$, as, si
1f) $(u, v) = (1/4, 4)$, as, si
1g) $(r, s) = (-2, -2)$, us, sa; $(r, s) = (1, 1)$, as, ssi
1h) $(x, y) = (0, 0)$, id
1i) $(x, y) = (0, 1)$, us, sso
1j) $(u, v) = (1, 1)$, us, sso
1k) $(x, y) = (0, 0)$, us, sa; $(x, y) = (c/d, a/b)$, id
1l) $(S, P) = (0, 0)$ us, sa; $(S, P) = (2, 0)$, us, sa; $(S, P) = (1, 1)$, as, ssi
1m) $(S, I) = (1, 0)$, as, si; $(S, I) = (2, -1/2)$, us, sa
1n) $(r, s) = (1, -3)$, as, si; $(r, s) = (-1, -2)$, us, sa

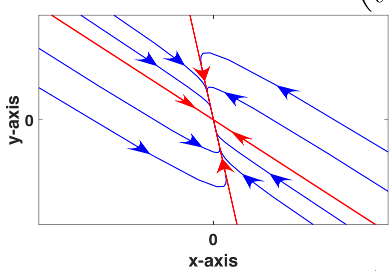
2a) (i) $u = v = 0$, (ii) $\begin{pmatrix} u' \\ v' \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$



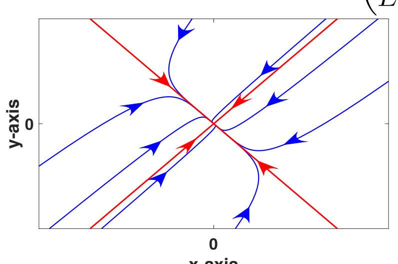
2b) (i) $u = 1$, $v = -1$, (ii) $\begin{pmatrix} u' \\ v' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} u-1 \\ v+1 \end{pmatrix}$



2c) (i) $u = 1/4$, $v = 4$, (ii) $\begin{pmatrix} u' \\ v' \end{pmatrix} = \begin{pmatrix} -16 & -2 \\ 16 & 1 \end{pmatrix} \begin{pmatrix} u-1/4 \\ v-4 \end{pmatrix}$



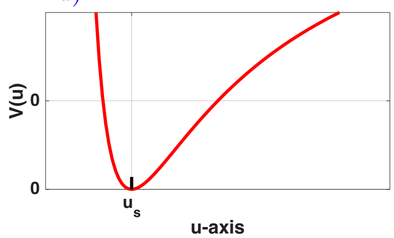
2d) (i) $S = 0$, $E = E_0$, (ii) $\begin{pmatrix} S' \\ E' \end{pmatrix} = \begin{pmatrix} -2E_0 & -1 \\ -2E_0 & -2 \end{pmatrix} \begin{pmatrix} S \\ E-E_0 \end{pmatrix}$



3a) $(x, y) = (a, a^3)$, us; $(x, y) = (-a, -a^3)$, as
3b) $(x, y) = (a, \cos(a))$, us
4) a) B, b) C, c) A, d) D
7a) $(S, T) = (N, 0)$, $(S, T) = (\beta/\alpha, (N - \beta/\alpha)/2)$
7b) $N < \beta/\alpha$
7c) $N > \beta/\alpha$

Section 5.3, pg 146
1a) $H = v^2 + 3e^{2u}/2 - 3u$

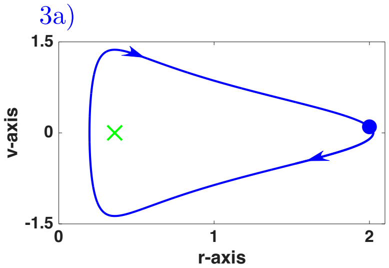
1b) $H = v^2/2 + \frac{1}{10}\ln(1+5u^2)$
1c) $H = 5v^2/2 + 7u^2/2 + \frac{3}{5}u^{10}$
1d) $H = v^2/2 + 4/3\,\left(u^2+1\right)^{3/2}$
2a) $K = v^2$, $V = \frac{3}{2}e^{2u} - 3u - \frac{3}{2}$
2b) $K = \frac{1}{2}v^2$, $V = \frac{1}{10}\ln(1+5u^2)$
2c) $K = \frac{1}{2}v^2$, $V = \frac{7}{2}u^2 + \frac{3}{5}u^{10}$
2d) $K = \frac{1}{2}v^2$, $V = \frac{4}{3}(1+u^2)^{3/2} - \frac{4}{3}$
3b) yes, $T = 2\pi$
3c) no
3d) $v = 1$, $u = t$
4a) one
4d)



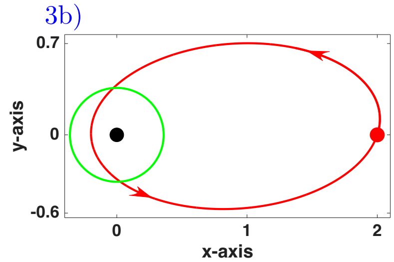
5a) $2v^2 + 2u^2 + u^4 = 3$
5b) $u = v = 0$
5c) clockwise
5d) $\sqrt{3/2}$
5e) -1
5f) $2\sqrt{2}\int_{-1}^{1}[(3+u^2)(1-u^2)]^{-1/2}du$
6a) $\frac{1}{2}v^2 + e^{-2u} - 2e^{-u} = e^{-2} - 2e^{-1}$
6b) $u = v = 0$
6c) clockwise
6d) $\sqrt{2}(1-e^{-1})$
6e) $-\ln(2-e^{-1})$
6f) $T = 2\sqrt{2}\int_{-\ln(2-e^{-1})}^{1}[(1-e^{-1})^2 - (1-e^{-u})^2]^{-1/2}du$
7c) $d = -a$
7d) $H = bv^2/2 - cu^2/2 + auv$

Section 5.4, pg 152
1a) no
1b) $v^2 + \alpha u^2 = \alpha u_0^2 + v_0^2$, where $\alpha = p^2 - k/m$
1c) this results in a $u = 0$ point
2a) $r_s = (mp^2/k)^{1/4}$; indeterminate
2b) $mv^2 + kr^2 + mp^2/r^2 = c$, where $c = mv_0^2 + kr_0^2[1 + (r_s/r_0)^4]$
2c) $r = r_s$ is where v takes its max/min values

3a)



3b)



Chapter 6
Section 6.1, pg 157

1a) $-\left(s-5\right)^{-1}$
1b) $(4+3\,s)/s^{2}$
1c) $\frac{2}{s^{2}}+\frac{7}{1+s}$
1d) $\frac{1}{s+2}-\frac{4}{(s-7)^{2}}$
1e) $8\,s^{-3}$
1f) $(9\,s^{2}-6\,s+2)/s^{3}$
1g) $4\,s^{-1}+8\,s^{-2}+8\,s^{-3}$
1h) $\frac{-10s}{s^{2}+64}$
1i) $\frac{5}{s}+\frac{8}{(s-3)^{2}+16}$
1j) $\frac{3}{s-1}+\frac{4s}{s^{2}+4}$
1k) $2\,\frac{s^{2}+3}{(s^{2}+9)(s^{2}+1)}$
1l) $\frac{50}{s(s^{2}+100)}$
2a) $6\,\frac{s}{(s^{2}+9)^{2}}$
2b) $6\,\frac{s^{2}-49}{(s^{2}+49)^{2}}$
2c) $2\,\frac{s\left(s^{2}-3\right)}{(s^{2}+1)^{3}}$
2d) $10\,\frac{s+2}{\left((s+2)^{2}+25\right)^{2}}$
3a) $\sum_{k=0}^{n} a_k \frac{k!}{s^{k+1}}$
3b) $\sum_{k=0}^{n} a_k/(s+k)$
3c) $\sum_{k=1}^{n} a_k \frac{k\pi}{k^{2}\pi^{2}+s^{2}}$

Section 6.2, pg 160

1a) $2/3\,\sin(3\,t)$
1b) $3t\,\mathrm{e}^{-4t}+5\,\mathrm{e}^{t}$
1c) $1/5\,\mathrm{e}^{t}-1/5\,\mathrm{e}^{-4\,t}$
1d) $\mathrm{e}^{-t}\cos(2\,t)$
1e) $1/4\,\mathrm{e}^{2\,t}+7/4\,\mathrm{e}^{-2\,t}$
1f) $1/3\,\mathrm{e}^{-t}\,(6\,\cos(3\,t)-5\,\sin(3\,t))$
1g) $-\cos(4t)+\cos(3t)$
1h) $\cosh(4t)-\cosh(t)$
1i) $\sin(2\,t)-\sin(3\,t)$
1j) $t\,\mathrm{e}^{t}+t^{2}\mathrm{e}^{-2t}+t^{3}\mathrm{e}^{3t}$
1k) $\sin(t)+\sinh(t)$
1l) $7\cos(t)-3t$
1m) $-2\,\mathrm{e}^{t}+1+\mathrm{e}^{2t}$
1n) $\mathrm{e}^{2t}-\mathrm{e}^{-t}\left(\sqrt{3}\,\sin(\sqrt{3}\,t)+\cos(\sqrt{3}\,t)\right)$
2a) $1-\cos(3t)$
2b) $1-\mathrm{e}^{-4t}\,(4t+1)$
2c) $-2\,\mathrm{e}^{t}+1+\mathrm{e}^{2t}$

Section 6.3, pg 163

1a) $-1+(s-4)\,Y$
1b) $4+(2\,s+7)\,Y$
1c) $s^{2}Y+5\,Y+2\,s+1$
1d) $\left(s^{2}+3\,s-2\right)Y+2s$
1e) $\left(4\,s^{2}+2\,s\right)Y+8\,s-2$
2a) $1/2\,\mathrm{e}^{t}-1/2\,\cos(t)-1/2\,\sin(t)$
2b) $1/2\,t\sin(t)$
2c) $\cos t(\sin t-2\,\cos t)+1+\mathrm{e}^{-t}$
2d) $-1/2\,\sin(t)+1/2\,\sinh(t)$
2e) $1/2\,t^{2}+\cos(t)-1$
2f) $-\cos(t)+1$

Section 6.4, pg 168

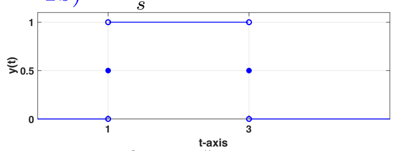
1a) $1+\mathrm{e}^{-t/2}$
1b) $-1/2\,\mathrm{e}^{-t}+\mathrm{e}^{-t/3}$
1c) $1/3\,\mathrm{e}^{-2\,t}-1/3\,\mathrm{e}^{t}$
1d) $2t\mathrm{e}^{3\,t}$
1e) $4-5\,\mathrm{e}^{t/5}$
1f) $-2\,\sin(t/2)-\cos(t/2)$
1g) $-\mathrm{e}^{t}\cos(t)$
1h) $-3\,\mathrm{e}^{-t}\sin(2\,t)$
2a) $4\,\mathrm{e}^{t}-\mathrm{e}^{-2\,t}-6\,t-3$
2b) $-1+\cos(2\,t)+2\,t^{2}$
2c) $\mathrm{e}^{t}-\sin(t)+\cos(t)-2$
2d) $1/2\,t^{2}$
2e) $1-1/2\,\mathrm{e}^{t}\cos(2\,t)-1/2\,\mathrm{e}^{-t}$
3a) $\int_0^t \ln(1+3\,\tau)\,\mathrm{e}^{-3\,t+3\,\tau}\,\mathrm{d}\tau$
3b) $\frac{1}{3}\int_0^t \sqrt{1+\tau}\sin(3\,t-3\,\tau)\,\mathrm{d}\tau$
3c) $-\frac{1}{5}\int_0^t \frac{\mathrm{e}^{-2\,t+2\,\tau}}{1+\tau}\,\mathrm{d}\tau+\frac{1}{5}\int_0^t \frac{\mathrm{e}^{t/2-\tau/2}}{1+\tau}\,\mathrm{d}\tau$
3d) $\frac{1}{2}\int_0^t \sin(1+\tau^{2})\mathrm{e}^{-t+\tau}\sin(2t-2\tau)\,\mathrm{d}\tau$
4a) $\int_0^t \ln(1+3\,\tau)\,\mathrm{e}^{-3\,t+3\,\tau}\,\mathrm{d}\tau+\mathrm{e}^{-3\,t}$
4b) $\cos(3\,t)+\frac{1}{3}\int_0^t \sqrt{1+\tau}\sin(3\,t-3\,\tau)\,\mathrm{d}\tau$
4c) $8/5\,\mathrm{e}^{-2\,t}+2/5\,\mathrm{e}^{t/2}-\frac{1}{5}\int_0^t \frac{\mathrm{e}^{-2\,t+2\,\tau}}{1+\tau}\,\mathrm{d}\tau+\frac{1}{5}\int_0^t \frac{\mathrm{e}^{t/2-\tau/2}}{1+\tau}\,\mathrm{d}\tau$

4d) $\mathrm{e}^{-t}\sin(2t) + \frac{1}{2}\int_0^t \sin(1+\tau^2)\mathrm{e}^{-t+\tau}\sin(2t-2\tau)\,\mathrm{d}\tau$

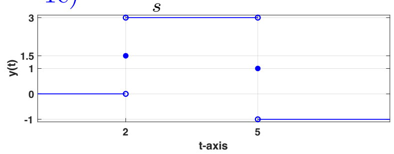
Section 6.5, pg 171

1a) $\frac{\mathrm{e}^{-6s}}{s}$

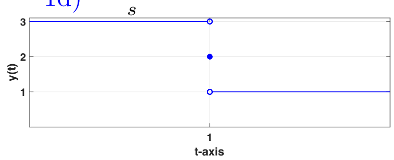
1b) $\frac{\mathrm{e}^{-s}-\mathrm{e}^{-3s}}{s}$



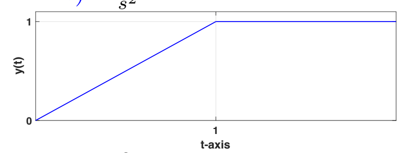
1c) $\frac{3\,\mathrm{e}^{-2s}-4\,\mathrm{e}^{-5s}}{s}$



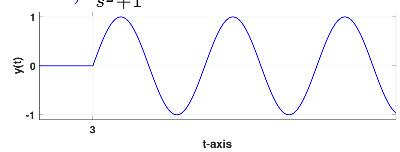
1d) $\frac{3-2\,\mathrm{e}^{-s}}{s}$



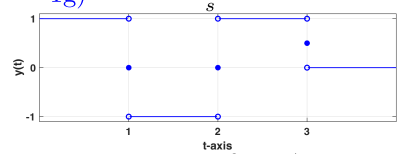
1e) $\frac{1-\mathrm{e}^{-s}}{s^2}$



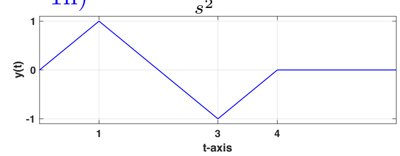
1f) $\frac{\mathrm{e}^{-3s}}{s^2+1}$



1g) $\frac{1-2\,\mathrm{e}^{-s}+2\,\mathrm{e}^{-2s}-2\,\mathrm{e}^{-3s}}{s}$



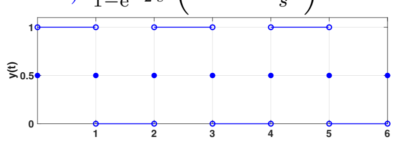
1h) $\frac{1-2\,\mathrm{e}^{-s}+2\,\mathrm{e}^{-3s}-\mathrm{e}^{-4s}}{s^2}$



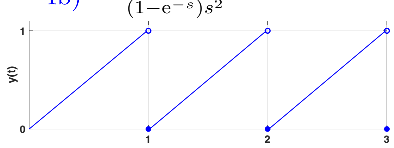
2a) $H(t-3)\mathrm{e}^{3-t}\cos(-9+3\,t)$
2b) $-1/2\,H(t-2)\,(t-2)\,(t-4)$
2c) $H(t-1)-H(t-2)+H(t-3)$

2d) $2-t+2\,t^2-7/6\,t^3$
2e) $H(t-5)t$
2f) $H(t-6)\,(5\,\cos(t-6)+\sin(t-6))$

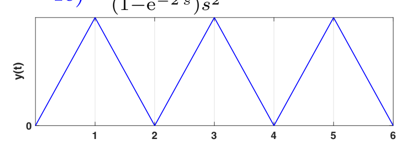
4a) $\frac{1}{1-\mathrm{e}^{-2s}}\left(s^{-1}-\frac{\mathrm{e}^{-s}}{s}\right)$



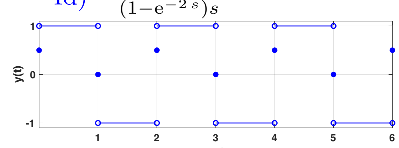
4b) $-\frac{\mathrm{e}^{-s}s+\mathrm{e}^{-s}-1}{(1-\mathrm{e}^{-s})s^2}$



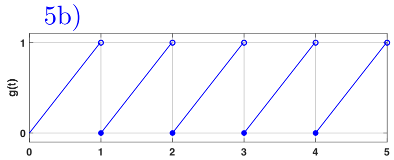
4c) $\frac{1+\mathrm{e}^{-2s}-2\,\mathrm{e}^{-s}}{(1-\mathrm{e}^{-2s})s^2}$



4d) $\frac{1+\mathrm{e}^{-2s}-2\,\mathrm{e}^{-s}}{(1-\mathrm{e}^{-2s})s}$

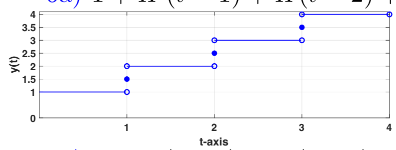


5a) 0, 0.1, 0.8, 0
5b)

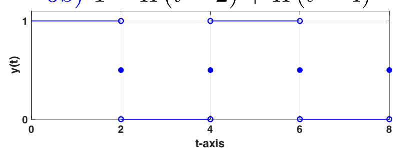


5c) $\frac{1}{(\mathrm{e}^s-1)s}$

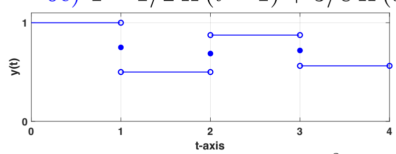
6a) $1 + H(t-1) + H(t-2) + H(t-3) + H(t-4) + \cdots$



6b) $1 - H(t-2) + H(t-4) - H(t-6) + H(t-8) + \cdots$



6c) $1 - 1/2\, H(t-1) + 3/8\, H(t-2) - \frac{5\, H(t-3)}{16} + \frac{35\, H(t-4)}{128} + \cdots$



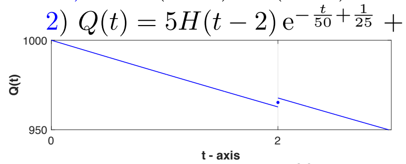
6d) $1 + t/2 - 1/16\, t^2 + \frac{t^3}{96} - \frac{5\, t^4}{3072} + + \cdots$

Section 6.6, pg 175

1a) $-(s-5)^{-1}$, $\mathrm{Re}(s) > 5$
1b) $8\, s^{-3}$, $\mathrm{Re}(s) > 0$
1c) $\frac{3s}{s^2+16}$, $\mathrm{Re}(s) > 0$
1d) $\frac{1}{(s+2)^2}$, $\mathrm{Re}(s) > -2$
2a) $\frac{\mathrm{e}^{-s}}{s^2}$
2b) $\frac{2-2\,\mathrm{e}^{-s}}{s^2}$
2c) $\frac{-(3s+1)\mathrm{e}^{-3s}+(s+1)\mathrm{e}^{-s}}{s^2}$
2d) $\frac{\mathrm{e}^{-2s}}{s}$
2e) $\frac{1-\mathrm{e}^{-4s\pi}}{s^2+1}$
2f) $\frac{4-4\,\mathrm{e}^{-3s}}{s}$

Section 6.7, pg 181

1a) $\mathrm{e}^{-4\,t} + 3/4\, H(t-1)\left(1 - \mathrm{e}^{-4\,t+4}\right)$
1b) $\left(-1 + \mathrm{e}^{t/2-2}\right) H(4-t) - \mathrm{e}^{t/2-2}$
1c) $2\, H(t-3)\mathrm{e}^{-t+3} - \mathrm{e}^{-t}$
1d) $-1/2\, H(t-2) + 1/2\, (1 - H(2-t))\, \mathrm{e}^{4\,t-8} + \mathrm{e}^{4\,t-4}\, (H(1-t) - 1)$
1e) $1/10\, H(t-5)\left(-5 + 3\,\mathrm{e}^{-2\,t+10} + 2\,\mathrm{e}^{3\,t-15}\right)$
1f) $3/2\, H(t-4)\,(\sin(t-4))^2 - 3/2\, H(t-2)\,(\sin(t-2))^2$
1g) $3/4\, H(t-1)\left(-1 + \mathrm{e}^{4\,t-4}\right)$
1h) $-2\, H(t-2)\sin(t-2) + H(t-3)\sin(t-3)$
2) $Q(t) = 5H(t-2)\,\mathrm{e}^{-\frac{t}{50}+\frac{1}{25}} + 950\,\mathrm{e}^{-\frac{t}{50}} + 50$



3a) $P' = 2P - 500\sum_{i=1}^{\infty}\delta(t-i)$, $P(0) = 100$
3b) $P = 100\,\mathrm{e}^{2t} - 500\,\mathrm{e}^{2t-2}\left(H(t-1) + \mathrm{e}^{-2}H(t-2) + \mathrm{e}^{-4}H(t-3) + \cdots\right)$
3c) in the 3d
4a) $2v' = -20 - v/2 + 70\delta(t-10)$, $v(0) = 0$
4b) $v = 35H(t-10)\,\mathrm{e}^{-\frac{t}{4}+\frac{5}{2}} - 40 + 40\,\mathrm{e}^{-\frac{t}{4}}$
4c) $x = 1160 - 40t - 160\,\mathrm{e}^{-\frac{t}{4}} + 140H(t-10)\,(1 - \mathrm{e}^{-\frac{t}{4}+\frac{5}{2}})$
5a) $T' = -k(T - 350 + 200H(t-120) - 200H(t-180))$, $T(0) = 70$
5b) $T = -200H(t-120)\left(1 - \mathrm{e}^{-k(t-120)}\right) + 200H(t-180)\left(1 - \mathrm{e}^{-k(t-180)}\right) - 280\,\mathrm{e}^{-kt} + 350$

5c) about 230 minutes

Section 6.8, pg 186

1a) $\begin{bmatrix} \frac{18\,e^{-3\,t}}{5} + 2/5\,e^{2\,t} \\ -6/5\,e^{-3\,t} + 1/5\,e^{2\,t} \end{bmatrix}$

1b) $\begin{bmatrix} 7/4\,e^{t/2} + 9/4\,e^{-t/2} \\ 7/2\,e^{t/2} - 9/2\,e^{-t/2} \end{bmatrix}$

1c) $\begin{bmatrix} 5/2\,e^{2\,t} + 3/2\,e^{4\,t} \\ -5/2\,e^{2\,t} + 3/2\,e^{4\,t} \end{bmatrix}$

1d) $\begin{bmatrix} \frac{13}{5} + 7/5\,e^{5\,t} \\ -\frac{26}{5} + \frac{21\,e^{5\,t}}{5} \end{bmatrix}$

1e) $\begin{bmatrix} 4\,e^{2\,t} \\ -e^{2\,t} - 4\,e^{2\,t}t \end{bmatrix}$

1f) $\begin{bmatrix} -\frac{1}{2}\,e^{t}\sin(2\,t) + 4\,e^{t}\cos(2\,t) \\ -e^{t}\cos(2\,t) - 8\,e^{t}\sin(2\,t) \end{bmatrix}$

2a) $\begin{bmatrix} \frac{2\,e^{2t}}{3} - \frac{8\,e^{-t}}{3} - 3t + 2 \\ \frac{4\,e^{2t}}{3} + \frac{8\,e^{-t}}{3} - 4 \end{bmatrix}$

2b) $\begin{bmatrix} 3t^2 \\ -6t^2 - 4t \end{bmatrix}$

2c) $\begin{bmatrix} e^{2t} - 2t - 1 \\ e^{2t} - e^{2t}t - t - 1 \end{bmatrix}$

2d) $\begin{bmatrix} -2\,e^{t}\cos(t) + e^{t}\sin(t) + t + 2 \\ e^{t}\sin(t) + 3\,e^{t}\cos(t) - 4t - 3 \end{bmatrix}$

3a) $\frac{1}{s^2+s-6}\begin{bmatrix} s & 6 \\ 1 & s+1 \end{bmatrix}$

3b) $\frac{1}{s^2-1/4}\begin{bmatrix} s & 1/4 \\ 1 & s \end{bmatrix}$

3c) $\frac{1}{(s-2)^2}\begin{bmatrix} s-2 & 0 \\ -1 & s-2 \end{bmatrix}$

3d) $\frac{1}{s^2-2\,s+5}\begin{bmatrix} s-1 & -4 \\ 1 & s-1 \end{bmatrix}$

4b) $Y_1 = \frac{10a+10s}{s(2a+s)}$, $a = 1/50$

4c) $y_1 = 5 + 5\,e^{-\frac{t}{25}}, y_2 = -5\,e^{-\frac{t}{25}} + 5$

4d) 5, 5

5a) $y_1' = -\frac{1}{20}y_1 + \frac{1}{50}y_2 + 6$

5b) $Y_1 = \frac{200(250s^2+155s+3)}{(50s+3)(100s+1)s}$

5c) $y_1 = -118\,e^{-\frac{t}{100}} - 72\,e^{-\frac{3t}{50}} + 200$, $y_2 = -236\,e^{-\frac{t}{100}} + 36\,e^{-\frac{3t}{50}} + 200$

5d) 200, 200

6b) $U_1 = (s^2+2)/(s^4+5s^2+2)$

6c) $u_1 = \frac{\sin(t)}{3} + \frac{\sin(2t)}{3}$, $u_2 = \frac{2\sin(t)}{3} - \frac{\sin(2t)}{3}$

7a) $v' = -f/m,\ f' = kv - kf/c$, $v(0) = 0,\ f(0) = ku_0$

7b) $V = -1/(s^2 + s/c + 1)$

7c) $v = -\frac{2\sqrt{3}\,e^{-\frac{t}{2}}\sin\left(\frac{\sqrt{3}\,t}{2}\right)}{3}$

7d) $u = e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}\,t}{2}\right) + \frac{e^{-\frac{t}{2}}\sqrt{3}\sin\left(\frac{\sqrt{3}\,t}{2}\right)}{3}$

7e) $c > 1/2$

Chapter 7

Section 7.2, pg 194

1a) $u(x) = \frac{e^{4x} - e^{-4x}}{e^4 - e^{-4}}$

1b) $u(x) = \frac{e^{-3x} - e^{3x}}{e^3 + e^{-3}}$

1c) $u(x) = \frac{e^x \sin(2x)}{e^2 \sin(4)}$

1d) $u(x) = \frac{-5\,e^2 + 5\,e^{2-x} + 5\,e^x - 5}{e^2 + 1}$

1e) $u(x) = -2\,e^{-x} + x^2 - 2x + 2\,e^{-1}$

1f) $u(x) = \frac{-e^{2x} + e^{4x}}{e^{4\pi} - e^{2\pi}} - \sin(4x)$

3a) $u_n = b_n \sin\left[\frac{\pi}{2}(2n-1)x\right],$ with $\lambda_n = -[\frac{\pi}{2}(2n-1)]^2$

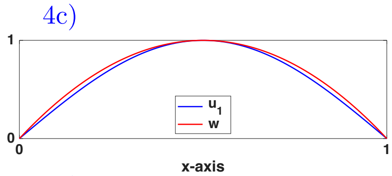
3b) $u_0 = b_0$, with $\lambda_0 = 0$; and $u_n = b_n \cos(n\pi x/4),$ with $\lambda_n = -(n\pi/4)^2$

3c) $u = be^{-\lambda x/2} \sin(\pi x/4),$ with $\lambda = \pm\sqrt{4 - (\pi/2)^2}$

3d) $u_n = b_n e^{-x} \sin(n\pi x),$ with $\lambda_n = -1 - (n\pi)^2$

3e) $u_0 = b_0$, $\lambda_0 = 0$; $u_n = a_n \sin(2\pi n x) + b_n \cos(2\pi n x),$ with $\lambda_n = 4\pi^2 n^2$

4c)



4d) -10

Section 7.3, pg 200

1a) $-4\,e^{-75\,\pi^2 t} \sin(5\,\pi\,x)$

1b) $6\,e^{-363\,\pi^2 t} \sin(11\,\pi\,x)$

1c) $e^{-3\,\pi^2 t} \sin(\pi\,x) + 8\,e^{-48\,\pi^2 t} \sin(4\,\pi\,x) - 10\,e^{-147\,\pi^2 t} \sin(7\,\pi\,x)$

1d) $-e^{-27\,\pi^2 t} \sin(3\,\pi\,x) + 7\,e^{-192\,\pi^2 t} \sin(8\,\pi\,x) + 2\,e^{-675\,\pi^2 t} \sin(15\,\pi\,x)$

1e) $2\,e^{-27\,\pi^2 t} \sin(3\,\pi\,x) + 2\,e^{-3\,\pi^2 t} \sin(\pi\,x)$

2a) $\sum_{n=1}^{\infty} \frac{2}{n\pi}(1 + (-1)^{n+1}) e^{-n^2\pi^2 t} \sin(1/2\,n\,\pi\,x)$

2b) $\sum_{n=1}^{\infty} \frac{4}{n\pi}(1 + 2(-1)^{n+1}) e^{-n^2\pi^2 t} \sin(n\,\pi\,x/2)$

2c) $-\frac{4}{3\pi} e^{-\pi^2 t} \sin(\pi\,x/2) + \sum_{n=3}^{\infty} \frac{2n}{\pi\,(n^2-4)}(1 + (-1)^{n+1}) e^{-n^2\pi^2 t} \sin(n\,\pi\,x/2)$

2d) $\sum_{n=1}^{\infty} \frac{2}{n\pi} (\cos(n\,\pi/2) - 1)\, e^{-n^2\pi^2 t} \sin(n\,\pi\,x/2)$

2e) $\sum_{n=1}^{\infty} \frac{2}{n\pi} (-2\,(-1)^n + \cos(n\,\pi 6) + 1)\, e^{-n^2\pi^2 t} \sin(n\,\pi\,x/2)$

3) $\sum_{n=1}^{\infty} \frac{30}{n\pi} (-1)^n\, e^{-n^2\pi^2 t} \sin(n\,\pi\,x/3)$

4) $\sum_{n=1}^{\infty} \frac{2}{n\pi}(\cos(n\,\pi/4) - \cos(3n\,\pi/4))\, e^{-5n^2\pi^2 t/2} \sin(n\,\pi\,x/2)$

5a) $\sum_{n=1}^{\infty} b_n e^{-k_n^2 t} \sin(k_n x),\ k_n = \pi(2n-1)/2$

5b) $\sum_{n=1}^{\infty} b_n e^{-4k_n^2 t} \cos(k_n x),\ k_n = \pi(2n-1)/2$

5c) $\sum_{n=1}^{\infty} b_n e^{-k_n^2(t + t^2/2)} \sin(k_n x),\ k_n = n\pi$

5d) $\sum_{n=1}^{\infty} b_n e^{-k_n^2 t + e^{-t}} \sin(k_n x),\ k_n = n\pi$

6a) $u = 3e^{-k_1^2 t} \sin(k_1 x) - 7e^{-k_5^2 t} \sin(k_5 x)$

6b) $u = -5e^{-4k_2^2 t}\cos(k_2 x) - 2e^{-4k_6^2 t}\cos(k_6 x)$

6c) $u = 14e^{-k_{10}^2(t+t^2/2)}\sin(k_{10}x) + 30e^{-k_{18}^2(t+t^2/2)}\sin(k_{18}x)$

6d) $u = -24e^{(-k_3^2 t + e^{-t}-1)}\sin(k_3 x) - 12e^{(-k_{15}^2 t+e^{-t}-1)}\sin(k_{15}x)$

7a) $(1+x)F'' = \lambda F$, $7G' - tG = \lambda G$

7b) $r^2R'' + rR' = \lambda R$, $\Theta'' = -\lambda\Theta$

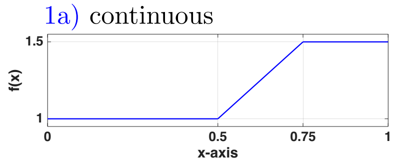
7c) $(e^x F')' = \lambda(1+x^2)F$, $G' = \lambda G$

7d) $Z'' + 3zZ' = \lambda Z$, $Y'' + 9Y = \lambda Y$

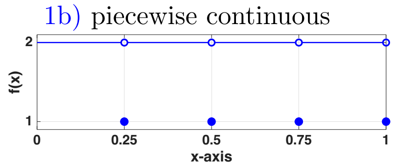
7e) $(F'/F)^2 = \lambda$, $(G'/G)^2 = e^{-t} - \lambda$

Section 7.4, pg 212

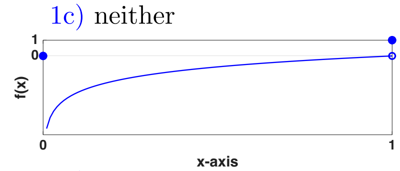
1a) continuous



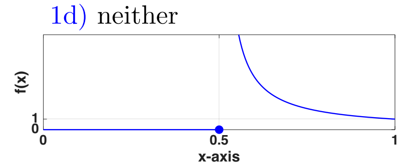
1b) piecewise continuous



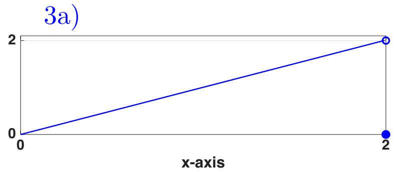
1c) neither



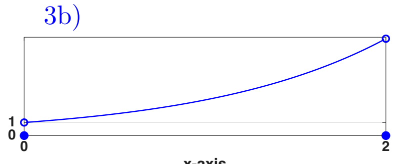
1d) neither



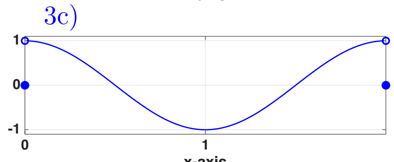
3a)



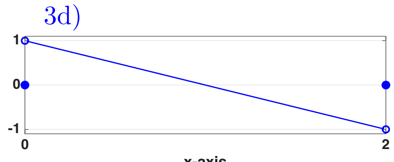
3b)



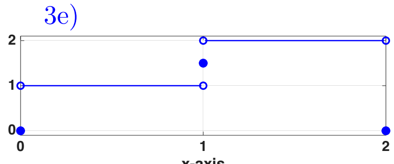
3c)



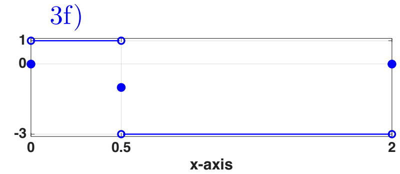
3d)



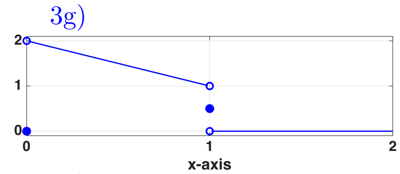
3e)



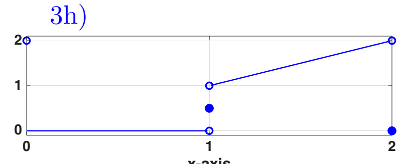
3f)



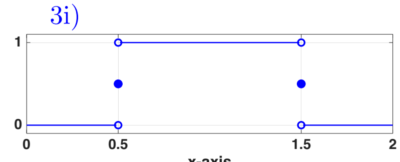
3g)



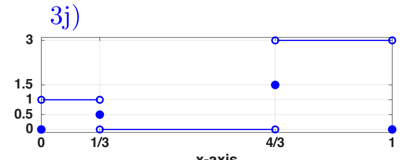
3h)



3i)



3j)



4a) $\sum_{n=1}^{\infty} \frac{4}{n\pi} (-1)^{n+1} \sin(n\,\pi\,x/2)$

4b) $\sum_{n=1}^{\infty} \frac{2n\pi}{n^2\pi^2+4} \left(1-(-1)^n\, e^2\right) \sin(n\,\pi\,x/2)$

4c) $$\sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} 4\,\frac{n\,\sin(n\,\pi\,x/2)}{\pi\,(n^2-4)}$$

4d) $$\sum_{\substack{n=1 \\ n \text{ even}}}^{\infty} \frac{4\sin(n\pi x/2)}{n\pi}$$

4e) $\sum_{n=1}^{\infty} \frac{1}{n\pi} \left(-4\,(-1)^n + 2\cos(n\pi/2) + 2\right) \sin(n\pi x/2)$

4f) $\sum_{n=1}^{\infty} \frac{1}{n\pi} \left(6\,(-1)^n - 8\,\cos(n\,\pi/4) + 2\right) \sin(n\,\pi\,x/2)$

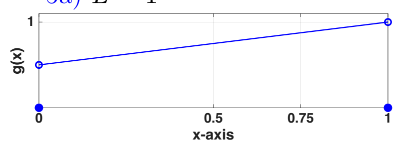
4g) $\sum_{n=1}^{\infty} \frac{1}{n^2\pi^2} \left(-2\,n\,\pi\,\cos(n\,\pi/2) - 4\,\sin(n\,\pi/2) + 4n\pi\right) \sin(n\,\pi\,x/2)$

4h) $\sum_{n=1}^{\infty} \frac{2}{n^2\pi^2} \Big(2\,(-1)^{n+1}\,n\pi + n\pi\cos(n\pi/2) - 2\sin(n\pi/2)\Big) \sin(n\pi x/2)$

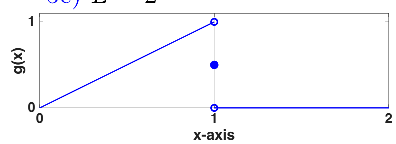
4i) $\sum_{n=1}^{\infty} \frac{1}{n\pi} \left(2\cos(n\pi/4) - 2\cos(3n\pi/4)\right) \sin(n\pi x/2)$

4j) $\sum_{n=1}^{\infty} \frac{1}{n\pi} \left(-6\,(-1)^n - 2\cos(n\pi/6) + 6\cos(2n\pi/3) + 2\right) \sin(n\pi x/2)$

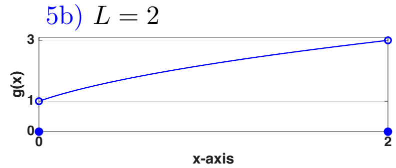
5a) $L = 1$



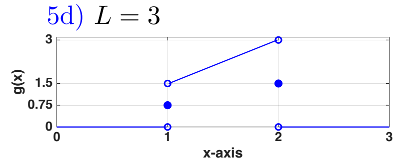
5c) $L = 2$



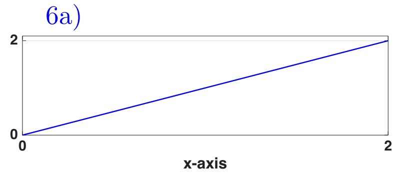
5b) $L = 2$



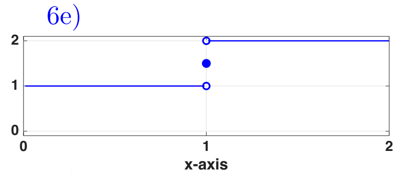
5d) $L = 3$



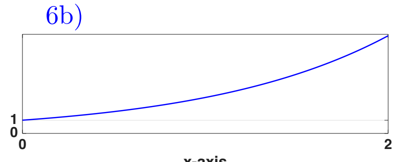
6a)



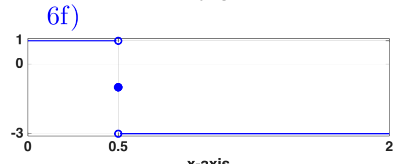
6e)



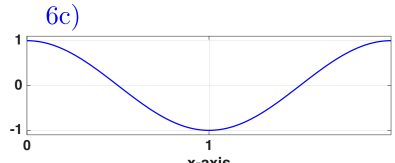
6b)



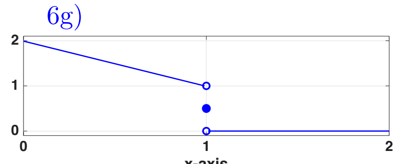
6f)



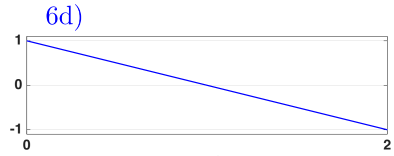
6c)



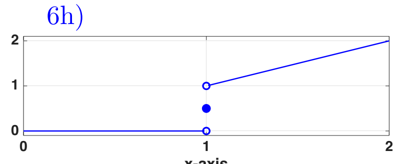
6g)



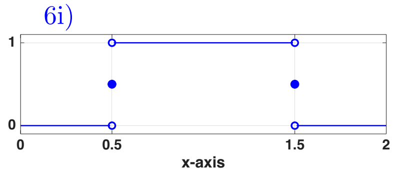
6d)



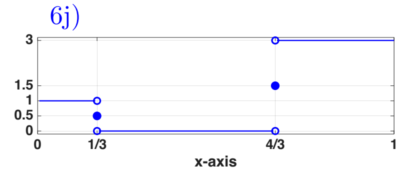
6h)



6i)



6j)



7a) $1 + \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} 8\cos(n\pi x/2)/(n^2\pi^2)$

7b) $-1/2 + 1/2\,\mathrm{e}^2 + \sum_{n=1}^{\infty} 4\,\frac{\left((-1)^n \mathrm{e}^2 - 1\right)\cos(n\pi x/2)}{n^2\pi^2+4}$

7c) $\cos(\pi x)$

7d) $\sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} 8\cos(n\pi x/2)/(n^2\pi^2)$

7e) $\frac{3}{2} - \sum_{n=1}^{\infty} \frac{2}{n\pi}\sin(n\pi/2)\cos(n\pi x/2)$

7f) $-2 + \sum_{n=1}^{\infty} \frac{8\sin\left(\frac{n\pi}{4}\right)\cos\left(\frac{n\pi x}{2}\right)}{n\pi}$

7g) $\frac{3}{4} + 2\sum_{n=1}^{\infty} \frac{\left(n\pi\sin\left(\frac{n\pi}{2}\right) - 2\cos\left(\frac{n\pi}{2}\right) + 2\right)\cos\left(\frac{n\pi x}{2}\right)}{n^2\pi^2}$

7h) $\frac{3}{4} + \sum_{n=1}^{\infty} \left(-\frac{2\left(n\pi\sin\left(\frac{n\pi}{2}\right) + 2\cos\left(\frac{n\pi}{2}\right) - 2(-1)^n\right)\cos\left(\frac{n\pi x}{2}\right)}{n^2\pi^2} \right)$

7i) $\frac{1}{2} + \sum_{n=1}^{\infty} \frac{\left(-2\sin\left(\frac{n\pi}{4}\right) + 2\sin\left(\frac{3n\pi}{4}\right)\right)\cos\left(\frac{n\pi x}{2}\right)}{n\pi}$

7j) $\frac{7}{6} + \sum_{n=1}^{\infty} \frac{\left(2\sin\left(\frac{n\pi}{6}\right) - 6\sin\left(\frac{2n\pi}{3}\right)\right)\cos\left(\frac{n\pi x}{2}\right)}{n\pi}$

9a) $1/3 + \sum_{n=1}^{\infty} 4\,\frac{(-1)^n\cos(n\pi x)}{n^2\pi^2}$

10a) $\sum_{n=1}^{\infty} -2\,\frac{(-1)^n\sin(n\pi x)}{n\pi}$

11 $g(x) = \begin{cases} x & \text{if } \ 0 \le x < 1/2 \\ 0 & \text{if } \ x = 1/2 \\ 1-x & \text{if } \ 1/2 < x \le 1 \end{cases}$

12a) for $x \ne 1/4$, $g = 1 - H(x - 1/4)$

12b) $g'(x) = -\delta(x - 1/4)$

Section 7.5, pg 219

1a) $\cos(12\pi t)\sin(3\pi x)$

1b) $-\frac{1}{16\pi}\sin(32\pi t)\sin(8\pi x)$

1c) $-\cos(4\pi t)\sin(\pi x) + 4\cos(12\pi t)\sin(3\pi x) - \frac{3\sin(20\pi t)\sin(5\pi x)}{20\pi}$

1d) $5\cos(28\pi t)\sin(7\pi x) + \frac{\sin(32\pi t)\sin(8\pi x)}{16\pi} + \frac{\sin(48\pi t)\sin(12\pi x)}{16\pi}$

1e) $\cos(12\pi t)\sin(3\pi x) + \cos(4\pi t)\sin(\pi x) - \frac{\sin(32\pi t)\sin(8\pi x)}{16\pi}$

1f) $3\cos(8\pi t)\sin(2\pi x) - \frac{\sin(36\pi t)\sin(9\pi x)}{24\pi} + \frac{3\sin(20\pi t)\sin(5\pi x)}{40\pi}$

2a) $\sum_{n=1}^{\infty} \left(a_n\cos(k_n t) + b_n\sin(k_n t)\right)\sin(k_n x)$, $k_n = (2n-1)\pi/2$

2b) $\sum_{n=1}^{\infty} \left(a_n\cos(2k_n t) + b_n\sin(2k_n t)\right)\cos(k_n x)$, $k_n = (2n-1)\pi/2$

2c) $a + bt + \sum_{n=1}^{\infty} \left(a_n\cos(n\pi t) + b_n\sin(n\pi t)\right)\left(A_n\cos(2n\pi x) + B_n\sin(2n\pi x)\right)$

2d) $e^{-t/2}\sum_{n=1}^{\infty} \left(a_n\cos(\omega_n t) + b_n\sin(\omega_n t)\right)\sin(n\pi x)\big)$, $\omega_n = \sqrt{4n^2\pi^2 - 1}/2$

3) $\sum_{n=1}^{\infty} -2\,\frac{\left(-1+(-1)^n\right)\sin(2n\pi t)\sin(n\pi x)}{n^4\pi^4}$

Section 7.6, pg 222

1a) $1-2\,x+\sum_{n=1}^{\infty}-2\,\frac{(1+(-1)^n)\mathrm{e}^{-4\,n^2\pi^2t}\sin(n\,\pi\,x)}{n\,\pi}$

1b) $2-7\,x+\sum_{n=1}^{\infty}-14\,\frac{(-1)^n\mathrm{e}^{-4\,n^2\pi^2t}\sin(n\,\pi\,x)}{n\,\pi}$

1c) $-4+5\,x+\sum_{n=1}^{\infty}8\,\frac{\mathrm{e}^{-4\,n^2\pi^2t}\sin(n\,\pi\,x)}{n\,\pi}$

2a) $1-x$

2b) $-7+2x$

2c) $-1+3x$

2d) $Ae^{x}+Be^{-x}$, where $A=(2-e^{-3})/(e^3-e^{-3})$, $B=(e^3-2)/(e^3-e^{-3})$

2e) $A+Be^{x}$, where $A=(1+e^2)/(1-e^2)$, $B=2/(e^2-1)$

3) $1-x-\sum_{n=1}^{\infty}4\left(2\pi n+4\left(-1\right)^n-\pi\right)\mathrm{e}^{-\frac{(2n-1)^2\pi^2t}{16}}\sin\left(\frac{(2n-1)\pi x}{4}\right)/\left((2n-1)^2\,\pi^2\right)$

4) $-1+3\,x+\sum_{n=1}^{\infty}2\,\left(1+2\,(-1)^n\right)\mathrm{e}^{-n^2\pi^2\left(t+1/2\,t^2\right)}\sin\left(n\,\pi\,x\right)/\left(n\,\pi\right)$

5) $1-2\,x-7\,\cos\left(9\,\pi\,t\right)\sin\left(3\,\pi\,x\right)$

Section 7.7, pg 226

1a) $\frac{\left(4\,k_5\,\cos(t)+4\,\sin(t)-4\,k_5\,\mathrm{e}^{-k_5\,t}\right)\sin(5\,\pi\,x)}{{k_5}^2+1}$, where $k_5=100\pi^2$

1b) $-\frac{\mathrm{e}^{-k_3\,t}\left(-1+\mathrm{e}^{k_3\,t-2\,t}\right)\sin(3\,\pi\,x)}{k_3-2}$, where $k_3=36\pi^2$

1c) $\sum_{n=1}^{\infty}-1/2\,\frac{(-1+(-1)^n)\left(\mathrm{e}^{-4\,n^2\pi^2t}-1\right)\sin(n\,\pi\,x)}{n^3\pi^3}$

2a) $w=x(x^3-L^3)/(12D)$

2b) $Dv_{xx}=v_t$, $v(0,t)=v(L,t)=0$, $v(x,0)=g(x)-w(x)$

2c) $u=w+\sum_{n=1}^{\infty}b_n\exp(-\lambda_n t)\sin(n\,\pi\,x/L)$, $b_n=\frac{2(-1)^{n+1}L^4\left(\pi^2n^2-2+2(-1)^n\right)}{\pi^5n^5D}$

3b) $w_n'=-r_nw_n$, $r_n=5+(n\pi/2)^2$

3c) $\sum_{n=1}^{\infty}\alpha_n e^{-r_nt}\sin(n\,\pi\,x/2)$

3d) $\sum_{n=1}^{\infty}4(-1)^{n+1}e^{-r_nt}\sin(n\,\pi\,x/2)/(n\pi)$

4 $\sum_{n=1}^{\infty}\alpha_n\exp(-\lambda_n^2(t+t^2/2))\sin(\lambda_n x)$, $\alpha_n=\frac{2}{n\pi}(1-(-1)^n)$, $\lambda_n=n\pi/3$

Section 7.8, pg 235

1a) $5\sinh(2\pi y)\sin(2\pi x)/\sinh(4\pi)$

1b) $-3\sinh(12\pi y)\sin(12\pi x)/\sinh(24\pi)$

1c) $\sinh(\pi y)\sin(\pi x)/\sinh(2\pi)-7\sinh(8\pi y)\sin(8\pi x)/\sinh(16\pi)$

1d) $-3\frac{\sinh(4\pi y)\sin(4\pi x)}{\sinh(8\pi)}-\frac{\sinh(7\pi y)\sin(7\pi x)}{\sinh(14\pi)}+6\frac{\sinh(20\pi y)\sin(20\pi x)}{\sinh(40\pi)}$

2a) $\frac{1}{2}r^3\cos(3\theta)$

2b) $1-3(r/2)^{15}\sin(15\theta)$

2c) $(r/2)\sin(\theta)+3(r/2)^5\cos(5\theta))$

2d) $4-2(r/2)^5\sin(5\theta)-4(r/2)^9\sin(9\theta)+8(r/2)^{14}\cos(14\theta)$

3a) $u_{xx}+u_{yy}$, $u(0,y)=u(x,0)=u(x,2)=0$ and $u(1,y)=g(y)$

3b) $\sum_{n=1}^{\infty}c_n\mathrm{sinh}(n\pi x/2)\sin(n\pi y/2)$

3c) $c_n\mathrm{sinh}(n\pi/2)=\int_0^2 g(y)\sin(n\pi y/2)dy$

3d) $7\sinh(3\pi x)\sin(3\pi y)/\sinh(3\pi)$

3e) $-2\sinh(2\pi x)\sin(2\pi y)/\sinh(2\pi)+8\sinh(7\pi x)\sin(7\pi y)/\sinh(7\pi)$

4a) $u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta}=0$, for $0<r<2$, $0<\theta<\pi/2$, $u(r,0)=u(r,\pi/2)=0$, $u(2,\theta)=f(\theta)$

4b) $\sum_{n=1}^{\infty} c_n r^{2n}\sin(2n\theta)$

4c) $-3(r/2)^4\sin(4\theta)$

4d) $9(r/2)^2\sin(2\theta)-5(r/2)^{14}\sin(14\theta)$

6 $u\big|_{\theta=-\pi}=u\big|_{\theta=\pi}\quad\text{and}\quad u_\theta\big|_{\theta=-\pi}=u_\theta\big|_{\theta=\pi}$